3.1 Basic Concepts of Probability and Counting

PROBABILITY EXPERIMENTS

When weather forecasters say that there is a 90% chance of rain or a physician says there is a 35% chance for a successful surgery, they are stating the likelihood, or *probability*, that a specific event will occur. Decisions such as "should you go golfing" or "should you proceed with surgery" are often based on these probabilities.

DEFINITION

A **probability experiment** is an action, or trial, through which specific results (counts, measurements, or responses) are obtained. The result of a single trial in a probability experiment is an **outcome.** The set of all possible outcomes of a probability experiment is the **sample space.** An **event** is a subset of the sample space. It may consist of one or more outcomes.

Rolling a die:

Sample Space: {1, 2, 3, 4, 5, 6}

Event: Roll an even number, {2, 4, 6}.

Outcome: Roll a 2, {2}.

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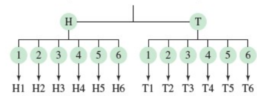
Ex 1)
A **tree diagram** gives a visual display of the outcomes of a probability experiment by using branches that originate from a starting point. It can be used to find the number of possible outcomes in a sample space as well as individual outcomes.

1a)

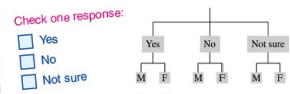
Tree Diagram for Coin and Die Experiment



From the tree diagram, you can see that the sample space has 12 outcomes.

{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

1b)
How many outcomes if you have to check a response and state if your male or female.



6 outcomes
Let Y = Yes, N = No, NS = Not sure, M = Male and F = Female.
Sample space = {YM, YF, NM, NF, NSM, NSF}

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Simple event: is an outcome or event that can't be broken into smaller components.

Ex) rolling one dice and getting a 5. Either it is or isn't a 5.

Not simple: rolling two die and getting the sum of 7. roll a 3 and a 4 or 2 and a 5 or 1 and a 6.

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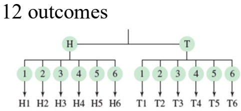
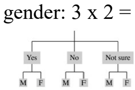
Tree diagram is time consuming, but shows all the different outcomes.

Ex 2)
Option #2. Counting Rule- multiply all the different number of choices.

Ex2a)
Rolling a die and flipping a coin:
6 numbers on the die and 2 sides of the coin
6 x 2 = 12 outcomes



Ex2b)
3 choices on the survey and 2 choices for gender: 3 x 2 = 6 outcomes



6 outcomes
Let Y = Yes, N = No, NS = Not sure, M = Male and F = Female.
Sample space = {YM, YF, NM, NF, NSM, NSF}

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Ex2c)
Using the Fundamental Counting Principle

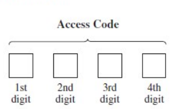
You are purchasing a new car. The possible manufacturers, car sizes, and colors are listed.

Manufacturer:	Ford, GM, Honda
Car size:	compact, midsize
Color:	white (W), red (R), black (B), green (G)

$3 \cdot 2 \cdot 4 = 24$ ways.

Ex2d)
Using the Fundamental Counting Principle

The access code for a car's security system consists of four digits. Each digit can be any number from 0 through 9.



How many access codes are possible when

1. each digit can be used only once and not repeated?
2. each digit can be repeated?
3. each digit can be repeated but the first digit cannot be 0 or 1?

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Ex2e)
How many license plates can you make when a license plate consists of

1. six (out of 26) alphabetical letters, each of which can be repeated?
2. six (out of 26) alphabetical letters, each of which cannot be repeated?
3. six (out of 26) alphabetical letters, each of which can be repeated but the first letter cannot be A, B, C, or D?

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Ex3)

TYPES OF PROBABILITY

The method you will use to calculate a probability depends on the type of probability. There are three types of probability: **classical probability, empirical probability,** and **subjective probability.** The probability that event E will occur is written as $P(E)$ and is read as "the probability of event E."

DEFINITION

Classical (or **theoretical**) **probability** is used when each outcome in a sample space is equally likely to occur. The classical probability for an event E is given by

$$P(E) = \frac{\text{Number of outcomes in event } E}{\text{Total number of outcomes in sample space}}.$$

Ex3a)

Finding Classical Probabilities

You roll a six-sided die. Find the probability of each event.

1. Event A: rolling a 3
2. Event B: rolling a 7
3. Event C: rolling a number less than 5

Ex3b)

Standard Deck of Playing Cards

Hearts	Diamonds	Spades	Clubs
A ♥	A ♦	A ♠	A ♣
K ♥	K ♦	K ♠	K ♣
Q ♥	Q ♦	Q ♠	Q ♣
J ♥	J ♦	J ♠	J ♣
10 ♥	10 ♦	10 ♠	10 ♣
9 ♥	9 ♦	9 ♠	9 ♣
8 ♥	8 ♦	8 ♠	8 ♣
7 ♥	7 ♦	7 ♠	7 ♣
6 ♥	6 ♦	6 ♠	6 ♣
5 ♥	5 ♦	5 ♠	5 ♣
4 ♥	4 ♦	4 ♠	4 ♣
3 ♥	3 ♦	3 ♠	3 ♣
2 ♥	2 ♦	2 ♠	2 ♣

You select a card from a standard deck of playing cards. Find the probability of each event.

1. Event D: Selecting the nine of clubs
2. Event E: Selecting a heart
3. Event F: Selecting a diamond, heart, club, or spade

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Ex4)

DEFINITION

Empirical (or **statistical**) **probability** is based on observations obtained from probability experiments. The empirical probability of an event E is the relative frequency of event E.

$$P(E) = \frac{\text{Frequency of event } E}{\text{Total frequency}} = \frac{f}{n}$$

%

Finding Empirical Probabilities

A company is conducting an online survey of randomly selected individuals to determine how often they recycle. So far, 2451 people have been surveyed. The frequency distribution shows the results. What is the probability that the next person surveyed always recycles? *(Adapted from Harris Interactive)*

Response	Number of times, f
Always	1054
Often	613
Sometimes	417
Rarely	196
Never	171
	$\Sigma f = 2451$

Ex4a)

Solution

The event is a response of "always." The frequency of this event is 1054. Because the total of the frequencies is 2451, the empirical probability of the next person always recycling is

$$P(\text{always}) = \frac{1054}{2451} \approx 0.430.$$

Ex4b)

Ages	Frequency, f
18 to 22	156
23 to 35	312
36 to 49	254
50 to 65	195
65 and over	58
	$\Sigma f = 975$

$$P(\text{age 23 to 35}) = \frac{312}{975} = 0.32.$$

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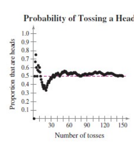
LAW OF LARGE NUMBERS

As an experiment is repeated over and over, the empirical probability of an event approaches the theoretical (actual) probability of the event.



Ex5)

The third type of probability is **subjective probability.** Subjective probabilities result from intuition, educated guesses, and estimates. For instance, given a patient's health and extent of injuries, a doctor may feel that the patient has a 90% chance of a full recovery. Or a business analyst may predict that the chance of the employees of a certain company going on strike is 0.25.

Classifying Types of Probability

Classify each statement as an example of classical probability, empirical probability, or subjective probability. Explain your reasoning.

1. The probability that you will get an A on your next test is 0.9.
2. The probability that a voter chosen at random will be younger than 35 years old is 0.3.
3. The probability of winning a 1000-ticket raffle with one ticket is $\frac{1}{1000}$.

Solution

1. This probability is most likely based on an educated guess. It is an example of subjective probability.
2. This statement is most likely based on a survey of a sample of voters, so it is an example of empirical probability.
3. Because you know the number of outcomes and each is equally likely, this is an example of classical probability.

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Ex6)

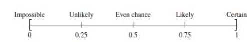
RANGE OF PROBABILITIES RULE

The probability of an event E is between 0 and 1, inclusive. That is,

$$0 \le P(E) \le 1.$$

When the probability of an event is 1, the event is certain to occur. When the probability of an event is 0, the event is impossible. A probability of 0.5 indicates that an event has an even chance of occurring or not occurring.

The figure below shows the possible range of probabilities and their meanings.



An event that occurs with a probability of 0.05 or less is typically considered unusual. Unusual events are highly unlikely to occur. Later in this course you will identify unusual events when studying inferential statistics.

It seems that no matter how strange an event is, somebody wants to know the probability that it will occur. The table below lists the probabilities that some intriguing events will happen. (Adapted from Life: The Odds)

Event	Probability
Being audited by the IRS	0.6%
Writing a *New York Times* best seller	0.0045
Winning an Academy Award	0.000087
Having your identity stolen	0.5%
Spotting a UFO	0.0000003

Which of these events is most likely to occur? Least likely?

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Ex7)

DEFINITION

The **complement of event E** is the set of all outcomes in a sample space that are not included in event E. The complement of event E is denoted by E' and is read as "E prime."

Think of them as opposite :
30% chance of rain, complement 70% chance it won't rain
10% chance you fail the exam, 90% chance you pass.

$$P(E) + P(E') = 1$$
$$P(E) = 1 - P(E')$$
$$P(E') = 1 - P(E)$$

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Ex 8)
Odds = # times an event occurs: # times an event doesn't occur

Bag of marbles with 5 green, 2 red, and 7 blue

Ex3a) Odds(green)

Ex 3b) Odds(blue)

Ex 4c) Odds(red)

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Odds against an Event

\# ways an event can't occur: # ways it can occur

Odds against rolling a 3 on 6 sided die =

Odds against drawing a heart in a deck of cards =

If given P(event) how do you find odds?

If given the $p(A) = \frac{3}{7}$ what's the odds(A)?

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