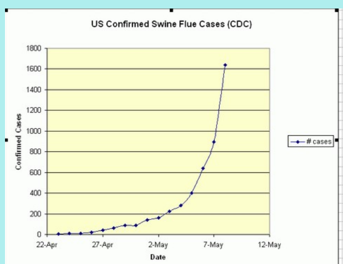
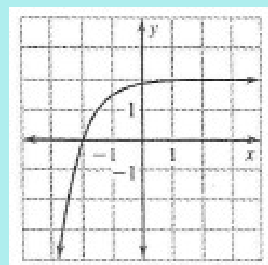
Exponential Growth and Decay
Chapter 8.8 Algebra I or Ch 8.2 Algebra II

- **When given $y = ab^x$ if $b>1$, then function represents a growth.**

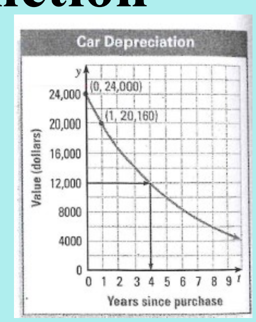
As the x's increase, so do the y's





ex) flu, savings account,population

- **When given $y = ab^x$ if $b<1$, then function represents a decay.**



As the x's increase, the y's decrease.

ex) price of a car, medication, radio active decay

Ex 1)

IDENTIFYING FUNCTIONS Tell whether the function represents *exponential growth* or *exponential decay.*

1 $f(x) = 4\left(\frac{3}{8}\right)^x$ 2. $f(x) = 10 \cdot 3^x$ *3. $f(x) = 8 \cdot 7^{-x}$ 4. $f(x) = 8 \cdot 7^x$

Try: 1. $f(x) = 8\left(\frac{2}{3}\right)^x$ 2. $f(x) = 3\left(\frac{4}{3}\right)^x$ 3. $f(x) = 5\left(\frac{1}{8}\right)^{-x}$

Real World Problems: $y = a \cdot b^x$

Ex1a) Suppose 10 animals are taken to an island, and then the population of these animals quadruples every year. Use the function $f(x) = 10 \cdot 4^x$. How many animals would there be after 6 years.

looking at the formula what is a, b, and x?

Ex1b Suppose 20 rabbits are taken to an island. The rabbit population then triples every half year. The function $f(x) = 20 \cdot 3^x$, where x is the number of half-year periods, models this situation. How many rabbits would there be after 2 years?

$f(x) = 20 \cdot 3^x$ looking at the formula what is a, b, and x?

$= 20 \cdot 3^4$ **In 2 years, there are 4 half years. Evaluate the function for $x = 4$.**

$= 20 \cdot 81$ **Simplify powers.**

$= 1620$ **Simplify.**

After two years, there would be 1620 rabbits.

In exponential functions $y = a \cdot b^x$

What represents a?
What represents b?
What represents x?

Try: Suppose 2 mice live in a barn. If the number of mice quadruples every 3 months, how many mice will be in the barn after 2 years?

1a. What is the exponential function?

1b. How many mice are there after 2 years?

GOAL 2 USING EXPONENTIAL GROWTH MODELS

When a real-life quantity increases by a fixed percent each year (or other time period), the amount y of the quantity after t years can be modeled by this equation:

Use when given % $y = a(1 + r)^t$

In this model, a is the initial amount and r is the percent increase expressed as a decimal. The quantity $1 + r$ is called the **growth factor**.

Ex2a)

In 1980 about 2,180,000 U.S. workers worked at home. During the next ten years, the number of workers working at home increased 5% per year.

a. Write a model giving the number w (in millions) of workers working at home t years after 1980. [redacted]

Ex2b)

In 1990 the cost of tuition at a state university was $4300. During the next 8 years, the tuition rose 4% each year.

a. Write a model that gives the tuition y (in dollars) t years after 1990.

[redacted]

TRY:

POPULATION The population of Winnemucca, Nevada, can be modeled by $P = 6191(1.04)^t$ where t is the number of years since 1990.

- What was the population in 1990?
- By what percent did the population increase each year?
- What is the population in 2014?

USING EXPONENTIAL DECAY MODELS

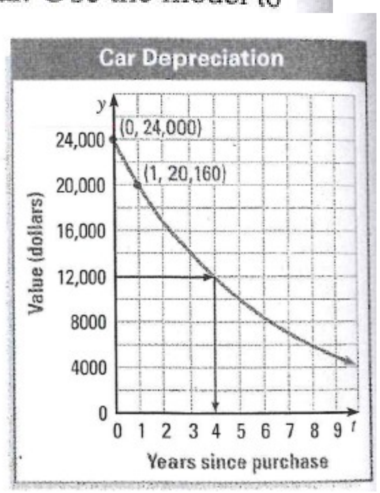
$$y = a(1 - r)^t$$

where a is the initial amount and r is the percent decrease expressed as a decimal. The quantity $1 - r$ is called the **decay factor**.

You buy a new car for $24,000. The value y of the car decreases by 16% each year.

Ex 3a.) Write an exponential decay model for the value of the car. Use the model to estimate the value after 2 years.

$$y = a(1 - r)^t$$
$$= \mathbf{24{,}000}(1 - 0.16)^t$$
$$= 24{,}000(0.84)^t$$
$$\approx \$16{,}934.$$



Ex3b)
An adult takes 400 milligrams of ibuprofen. Each hour, the amount of medicine in their person's system decreases by about 29%. How much milligrams are in the body's system after 4 hours?

COMPOUND INTEREST Exponential growth functions are used in real-life situations involving *compound interest.* Compound interest is interest paid on the initial investment, called the *principal,* and on previously earned interest. (Interes paid only on the principal is called *simple interest.*)

COMPOUND INTEREST

Consider an initial principal P deposited in an account that pays interest at annual rate r (expressed as a decimal), compounded n times per year. The amount A in the account after t years can be modeled by this equation:

$$A = P\left(1 + \frac{r}{n}\right)^{(nt)}$$

p = principal (starting amount)
n = number of times you get paid interest/year
r = rate as a decimal %/100
t= time (years or months/12)

Annual Interest Rate of x%

Compounded	Periods per Year	Interest Rate per Period
annually	1	x% every year
semi-annually	2	$\frac{x\%}{2}$ = 6 months
quarterly	4	$\frac{x\%}{4}$ = 3 months
monthly	12	$\frac{x\%}{12}$ = every month

$$A = P\left(1 + \frac{r}{n}\right)^{(nt)}$$

FINANCE You deposit $1000 in an account that pays 8% annual interest. Find the balance after 1 year if the interest is compounded with the given frequency.

a. annually **b.** quarterly **c.** daily

a. With interest compounded annually, the balance at the end of 1 year is:

$$A = 1000\left(1 + \frac{0.08}{1}\right)^{1 \cdot 1} \quad P = 1000,\ r = 0.08,\ n = 1,\ t = 1$$

$$= 1000(1.08)^1 \quad \text{Simplify.}$$

$$= 1080 \quad \text{Use a calculator.}$$

▶ The balance at the end of 1 year is $1080.

b. With interest compounded quarterly, the balance at the end of 1 year is:

$$A = 1000\left(1 + \frac{0.08}{4}\right)^{4 \cdot 1} \quad P = 1000,\ r = 0.08,\ n = 4,\ t = 1$$

$$= 1000(1.02)^4 \quad \text{Simplify.}$$

$$\approx 1082.43 \quad \text{Use a calculator.}$$

▶ The balance at the end of 1 year is $1082.43.

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ACCOUNT BALANCE You deposit \$500 in an account that pays 3% annual interest. Find the balance after 2 years if the interest is compounded with the given frequency.

a. annually

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

p = 500
r = .03
n=1
t=2

$500\left(1 + \frac{.03}{1}\right)^{(1\cdot 2)}$

\$530.45

b. quarterly

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

p = 500
r = .03
n=4
t=2

$500\left(1 + \frac{.03}{4}\right)^{(4\cdot 2)}$

\$530.80

c. daily

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

p = 500
r = .03
n=365
t=2

$500\left(1 + \frac{.03}{365}\right)^{(365\cdot 2)}$

\$530.92

TRY: Your parents deposit \$5000 into a certificate of deposit (CD) when you were born. They plan to leave it in for 18 years so you can put the money towards your college. If your CD rate remained 2.5% and the interest was compiled quarterly, how much money will you have when you turn 18?