

Ch 7.4 Hypothesis Testing for small sample sizes.

Assumptions

for testing claims about population means

- 1) The sample is a simple random sample.
- 2) The sample is **small** ($n \leq 30$).
- 3) The value of the population standard deviation σ is unknown.
- 4) The sample values come from a population with a distribution that is approximately normal.

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Test Statistic for a Student t -distribution

$$t = \frac{\bar{x} - \mu_x}{\frac{s}{\sqrt{n}}}$$

Critical Values

- ❖ Found in Table A-3
- ❖ Degrees of freedom (df) = $n - 1$
- ❖ Critical t values to the left of the mean are negative

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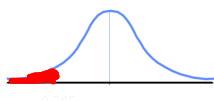
Ex1a) The Metro Bus Co. claims that the mean waiting time for a bus during rush hour is **less than 10 minutes**. A random sample of 20 waiting times has a mean of 8.5 minutes with a standard deviation of 2.1 minutes. At $\alpha = .01$, test the bus company's claim.

1. $H_0 = \mu \geq 10$ $H_1 = \mu < 10$

2. Find the test statistic: $\frac{8.5 - 10}{\frac{2.1}{\sqrt{20}}}$

3. Find the t critical value:

df = 19, $\alpha = .01$ 1 tailed test



4. Determine if reject or fail to reject:

Reject H_0 . There is sufficient evidence to show that waiting times are less than 10 minutes. The bus is late.

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Ex1b) A local group claims that the police issue at least 60 speeding tickets a day in their area. To prove their points, they randomly select 2 weeks. Their research yields the number of tickets issued for each day. Use $\alpha = .01$ to test the group's claim.

70 48 41 68 69 55 70 57 60 83 32 60 72 58

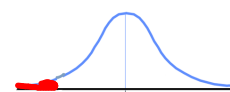
1. $H_0 = \mu \geq 60$ $H_1 = \mu < 60$

2. Find the test statistic:

$$\frac{60.2142 - 60}{\frac{13.42889}{\sqrt{14}}}$$

3. Find the t critical value:

1 tail, df = 13, $\alpha = .01$



4. Determine if reject or fail to reject:

Fail to reject H_0 . There is not enough evidence to show that the average number of tickets issued is greater than or equal to 60 tickets.

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TRY: A manufacturer claims that the mean lifetime of its bulbs is 1000 hours. A homeowner selects 25 bulbs and finds the mean life is 980 hours with $s = 80$ hours. Test the manufacturer's claim with $\alpha = .05$

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P-Value Method

- ❖ Table A-3 includes only selected values of α
- ❖ Specific P -values usually cannot be found
- ❖ Use Table to identify limits that contain the P -value
- ❖ Some calculators and computer programs will find exact P -values

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Degrees of freedom	α	0.05 (one tail) 0.1 (two tails)	0.025 (one tail) 0.05 (two tails)	0.01 (one tail) 0.02 (two tails)	0.005 (one tail) 0.01 (two tails)	0.0025 (one tail) 0.005 (two tails)	0.001 (one tail) 0.002 (two tails)
1		63.657	31.821	12.706	6.314	3.078	1.000
2		9.925	6.965	4.303	2.920	1.886	.816
3		5.841	4.541	3.182	2.353	1.638	.765
4		4.604	3.747	2.776	2.132	1.533	.741
5		4.032	3.365	2.571	2.015	1.476	.727
6		3.707	3.143	2.447	1.943	1.440	.719
7		3.500	2.998	2.365	1.895	1.415	.711
8		3.355	2.896	2.306	1.860	1.397	.706
9		3.250	2.819	2.262	1.833	1.383	.703
10		3.169	2.764	2.228	1.812	1.372	.700
11		3.106	2.718	2.201	1.796	1.363	.697
12		3.054	2.681	2.173	1.782	1.356	.696
13		3.012	2.650	2.160	1.771	1.350	.694
14		2.977	2.625	2.145	1.761	1.345	.692
15		2.947	2.602	2.132	1.753	1.341	.691
16		2.921	2.584	2.120	1.746	1.337	.690
17		2.898	2.567	2.110	1.740	1.333	.689
18		2.878	2.552	2.101	1.734	1.330	.688
19		2.861	2.540	2.093	1.729	1.328	.688
20		2.845	2.528	2.086	1.725	1.325	.687
21		2.831	2.518	2.080	1.721	1.323	.686
22		2.819	2.508	2.074	1.717	1.321	.686
23		2.807	2.500	2.069	1.714	1.320	.685
24		2.797	2.492	2.064	1.711	1.318	.685
25		2.787	2.485	2.060	1.708	1.316	.684
26		2.779	2.479	2.056	1.706	1.315	.684
27		2.771	2.473	2.052	1.703	1.314	.684
28		2.763	2.467	2.048	1.701	1.313	.683
29		2.756	2.462	2.045	1.699	1.311	.683
Large (z)		2.575	2.327	1.960	1.645	1.282	.675

Ex2a) If the test statistic is 2.567, n = 10, and it is a 2-tailed test, find the p-value.

.02 < p-value < .05

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Ex2b) A fast food claims the mean waiting times is less than 3.5 minutes. A random sample of 20 customers has a mean wait time of 3.7 minutes, s= .8 minutes and find the p-value to test the claim of the fast food company.

1. $H_0 =$ $H_1 =$ 2. Find the test statistic:

3. Find the p-value:

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