Ch 6.3 Finding population means from small samples

If 1) $n \leq 30$
2) The sample is a simple random sample.
3) The sample is from a normally distributed population.

Case 1 (σ is known): use $z_{\alpha/2}$ to determine the confidence interval.

Case 2 (σ is unknown): Use Student t distribution

☆ If data is skewed and n < 30 can't determine the interval.

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Student t Distribution

If the distribution of a population is essentially normal, then the distribution of

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

- is essentially a Student t Distribution for all samples of size n.
- is used to find critical values denoted by $t_{\alpha/2}$

Margin of Error E for Estimate of μ

Based on an Unknown σ and a Small Simple Random Sample from a Normally Distributed Population

Formula 6-2

$$E = t_{\alpha/2} \frac{s}{\sqrt{n}}$$

where $t_{\alpha/2}$ has n - 1 degrees of freedom

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Degrees of Freedom (df)

corresponds to the number of sample values that can vary after certain restrictions have imposed on all data values

df = n - 1

in this section

Ex)10 students have quiz scores that have a mean of 80. You can randomly pick 9 students scores, but the 10th quiz has to be a certain number to get the mean to = 80.

You can *freely* pick 9 scores. = df

Any #	Any #	Any #	Any #	Any #	Any #	Any #	Any #	Any #	Specific #

n = 10 ➡ df = 10 - 1 = 9

so that $\bar{x}$ = 80

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Confidence Interval for the Estimate of E

Based on an Unknown σ and a Small Simple Random Sample from a Normally Distributed Population

$$\bar{x} - E < \mu < \bar{x} + E$$

where $E = t_{\alpha/2} \frac{s}{\sqrt{n}}$

$t_{\alpha/2}$ found in Table A-3

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Table A-3 t Distribution

Degrees of freedom	.005 (one tail) .01 (two tails)	.01 (one tail) .02 (two tails)	.025 (one tail) .05 (two tails)	.05 (one tail) .10 (two tails)	.10 (one tail) .20 (two tails)	.25 (one tail) .50 (two tails)
1	63.657	31.821	12.706	6.314	3.078	1.000
2	9.925	6.965	4.303	2.920	1.886	.816
3	5.841	4.541	3.182	2.353	1.638	.765
4	4.604	3.747	2.776	2.132	1.533	.741
5	4.032	3.365	2.571	2.015	1.476	.727
6	3.707	3.143	2.447	1.943	1.440	.718
7	3.500	2.998	2.365	1.895	1.415	.711
8	3.355	2.896	2.306	1.860	1.397	.706
9	3.250	2.821	2.262	1.833	1.383	.703
10	3.169	2.764	2.228	1.812	1.372	.700
11	3.106	2.718	2.201	1.796	1.363	.697
12	3.054	2.681	2.179	1.782	1.356	.696
13	3.012	2.650	2.160	1.771	1.350	.694
14	2.977	2.625	2.145	1.761	1.345	.692
15	2.947	2.602	2.132	1.753	1.341	.691
16	2.921	2.584	2.120	1.746	1.337	.690
17	2.898	2.567	2.110	1.740	1.333	.689
18	2.878	2.552	2.101	1.734	1.330	.688
19	2.861	2.540	2.093	1.729	1.328	.688
20	2.845	2.528	2.086	1.725	1.325	.687
21	2.831	2.518	2.080	1.721	1.323	.686
22	2.819	2.508	2.074	1.717	1.321	.686
23	2.807	2.500	2.069	1.714	1.320	.685
24	2.797	2.492	2.064	1.711	1.318	.685
25	2.787	2.485	2.060	1.708	1.316	.684
26	2.779	2.479	2.056	1.706	1.315	.684
27	2.771	2.473	2.052	1.703	1.314	.684
28	2.763	2.467	2.048	1.701	1.313	.683
29	2.756	2.462	2.045	1.699	1.311	.683
Large (z)	2.575	2.327	1.960	1.645	1.282	.675

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Important Properties of the Student t Distribution

1. The Student t distribution is different for different sample sizes (see Figure 6-5 for the cases n = 3 and n = 12).
2. The Student t distribution has the same general symmetric bell shape as the normal distribution but it reflects the greater variability (with wider distributions) that is expected with small samples.
3. The Student t distribution has a mean of t = 0 (just as the standard normal distribution has a mean of z = 0).
4. The standard deviation of the Student t distribution varies with the sample size and is greater than 1 (unlike the standard normal distribution, which has a σ = 1).
5. As the sample size n gets larger, the Student t distribution gets closer to the normal distribution. For values of n > 30, the differences are so small that we can use the critical z values instead of developing a much larger table of critical t values. (The values in the bottom row of Table A-3 are equal to the corresponding critical z values from the standard normal distribution.)

Student t Distributions for n = 3 and n = 12

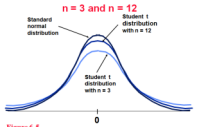

Figure 6-5

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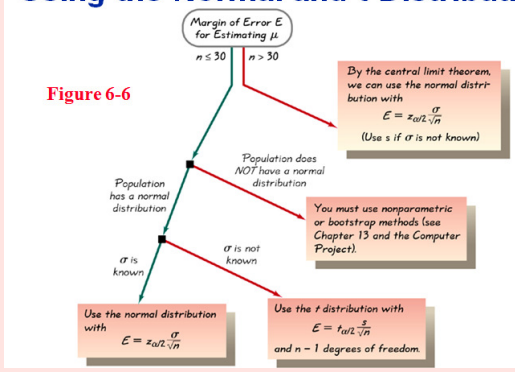
Using the Normal and *t* Distribution

Figure 6-6



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Example: A study of 12 Dodge Vipers involved in collisions resulted in repairs averaging $26,227 and a standard deviation of $15,873. Find the 95% interval estimate of μ, the mean repair cost for all Dodge Vipers involved in collisions. (The 12 cars' distribution appears to be bell-shaped.)

$\bar{x} = 26{,}227$

$s = 15{,}873$

$\alpha = 0.05$

$\alpha/2 = 0.025$

$n \leq 30$ use $t_{\alpha/2}$

$E = t_{\alpha/2}\frac{s}{\sqrt{n}}$

Table A-3 *t* Distribution

Degrees of freedom	.005 (one tail) .01 (two tails)	.01 (one tail) .02 (two tails)	.025 (one tail) .05 (two tails)
1	63.657	31.821	12.706
2	9.925	6.965	4.303
3	5.841	4.541	3.182
4	4.604	3.747	2.776
5	4.032	3.365	2.571
6	3.707	3.143	2.447
7	3.500	2.998	2.365
8	3.355	2.896	2.306
9	3.250	2.821	2.262
10	3.169	2.764	2.228
11	3.106	2.718	2.201
12	3.054	2.681	2.179
13	3.012	2.650	2.160
14	2.977	2.625	2.145

$\bar{x} - E < \mu < \bar{x} + E$

☆ [illegible] $< \mu <$ [illegible]

We are 95% confident that this interval contains the average cost of repairing a Dodge Viper.

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Ex1b) Create a confidence interval for salaries for airline pilots with 90% confidence: n = 25, $\bar{x}$ = $97,334, s = $17, 747

Table A-3 *t* Distribution

Degrees of freedom	.005 (one tail) .01 (two tails)	.01 (one tail) .02 (two tails)	.025 (one tail) .05 (two tails)	.05 (one tail) .10 (two tails)	.10 (one tail) .20 (two tails)	.25 (one tail) .50 (two tails)
21	2.831	2.518	2.080	1.721	1.323	.686
22	2.819	2.508	2.074	1.717	1.321	.686
23	2.807	2.500	2.069	1.714	1.320	.685
24	2.797	2.492	2.064	1.711	1.318	.685
25	2.787	2.485	2.060	1.708	1.316	.684
26	2.779	2.479	2.056	1.706	1.315	.684

$E = t_{\alpha/2}\frac{s}{\sqrt{n}}$

$\bar{x} - E < \mu < \bar{x} + E$

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Try:Find a 99% confidence interval for the mean heart rate while shoveling snow. If we are looking at 10 men, $\bar{x}$ = 174 and s = 15

Table A-3 *t* Distribution

Degrees of freedom	.005 (one tail) .01 (two tails)	.01 (one tail) .02 (two tails)
1	63.657	31.821
2	9.925	6.965
3	5.841	4.541
4	4.604	3.747
5	4.032	3.365
6	3.707	3.143
7	3.500	2.998
8	3.355	2.896
9	3.250	2.821
10	3.169	2.764
11	3.106	2.718

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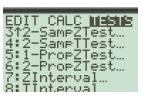
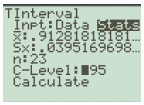
Using the graphing calculator:

Under stats you can go under test to compute the confidence intervals:

#8 is the t interval



High light data: when given the list of numbers and have entered them in L_1
High light stats: when you know the mean and standard deviation



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