

6-2

Estimating a Population Mean: Large Samples

❖ $n > 30$

The sample must have more than 30 values.

❖ Simple Random Sample

All samples of the same size have an equal chance of being selected.

Jan 27-8:32 AM

Definitions

❖ Estimator

a formula or process for using sample data to estimate a population parameter

❖ Estimate

a specific value or range of values used to approximate some population parameter

❖ Point Estimate

a single value (or point) used to approximate a population parameter

The sample mean \bar{x} is the best point estimate of the population mean μ .

Jan 27-8:33 AM

Definition

Confidence Interval

(or Interval Estimate)

a range (or an interval) of values **used to estimate** the true value of the population parameter

Lower # < population parameter < Upper #

As an example

Lower # < μ < Upper #

Jan 27-8:34 AM

Definition

Degree of Confidence

(level of confidence or confidence coefficient)

the probability $1 - \alpha$ (often expressed as the equivalent percentage value) that is the relative frequency of times the confidence interval actually does contain the population parameter, assuming that the estimation process is repeated a large number of times

usually 90%, 95%, or 99%

($\alpha = 10\%$), ($\alpha = 5\%$), ($\alpha = 1\%$)

Jan 27-8:36 AM

Interpreting a Confidence Interval

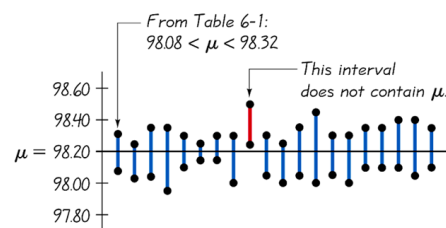
$$98.08 < \mu < 98.32$$

Correct: We are 95% confident that the interval from 98.08 to 98.32 actually does contain the true value of μ . This means that if we were to select many different samples of size 106 and construct the confidence intervals, 95% of them would actually contain the value of the population mean μ .

Wrong: There is a 95% chance that the true value of μ will fall between 98.08 and 98.32.

Jan 27-8:36 AM

Confidence Intervals from 20 Different Samples



Jan 27-8:37 AM

Definition

Critical Value

the number on the borderline separating sample statistics that are likely to occur from those that are unlikely to occur. The number $z_{\alpha/2}$ is a critical value that is a z score with the property that it separates an area $\alpha/2$ in the right tail of the standard normal distribution.

Jan 27-8:38 AM

The Critical Value $z_{\alpha/2}$

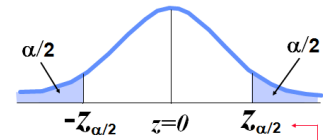


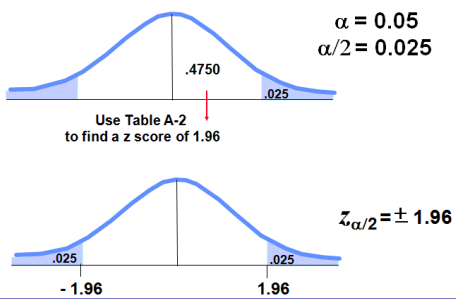
Figure 6-2

Found from Table A-2
(corresponds to area of $0.5 - \alpha/2$)

Jan 27-8:40 AM

Ex1) Find the critical Value for $z_{\alpha/2}$ that corresponds to the given degree of confidence.

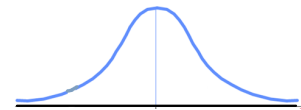
Finding $z_{\alpha/2}$ for 95% Degree of Confidence



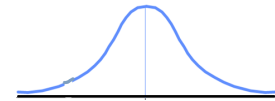
Jan 27-8:39 AM

Ex1) Find the critical Value for $z_{\alpha/2}$ that corresponds to the given degree of confidence.

b) 99%



c) 90%



Jan 27-8:45 AM

Degree of Confidence	α	Critical Value $\alpha/2$
90 %	0.10	1.645
95 %	0.05	1.96
99 %	0.01	2.575

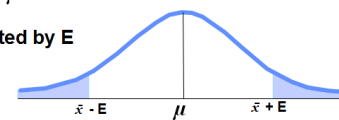
Jan 27-8:41 AM

Definition

Margin of Error

is the maximum likely difference observed between sample mean \bar{x} and true population mean μ .

denoted by E



$$\bar{x} - E < \mu < \bar{x} + E$$

lower limit upper limit

Jan 27-8:49 AM

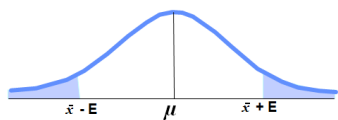
central limit theorem:

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

* since $n > 30$, we will use S for our standard deviation not μ .

Margin of Error

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \quad \text{Formula 6-1}$$



also called the **maximum error of the estimate**

Jan 27-8:50 AM

Confidence Interval (or Interval Estimate) for Population Mean μ

(Based on Large Samples: $n > 30$)

$$\bar{x} - E < \mu < \bar{x} + E$$

$$\mu = \bar{x} \pm E$$

$$(\bar{x} + E, \bar{x} - E)$$

Jan 27-9:09 AM

Procedure for Constructing a Confidence Interval for μ

(Based on a Large Sample: $n > 30$)

1. Find the **critical value** $z_{\alpha/2}$ that corresponds to the desired degree of confidence.
2. Evaluate the margin of error $E = z_{\alpha/2} \cdot \sigma / \sqrt{n}$.
If the population standard deviation σ is unknown, use the value of the sample standard deviation s provided that $n > 30$.
3. Find the values of $\bar{x} - E$ and $\bar{x} + E$. Substitute those values in the general format of the confidence interval: $\bar{x} - E < \mu < \bar{x} + E$
4. Round using the confidence intervals roundoff rules.

Jan 27-9:09 AM

Round-Off Rule for Confidence Intervals Used to Estimate μ

1. When using the **original set of data**, round the confidence interval limits to **one more decimal place** than used in original set of data.
★ if given a list of # and have to find everything to get your interval. No mean or standard deviation is given. When using a TI 84.
2. When the original set of data is unknown and only the **summary statistics** (n, \bar{x}, s) are used, round the confidence interval limits to **the same number of decimal places** used for the **sample mean**.

Jan 27-9:10 AM

Example: A study found the body temperatures of 106 healthy adults. The sample mean was 98.2 degrees and the sample standard deviation was 0.62 degrees. Find the margin of error E and the 95% confidence interval.

$n = 106$
 $\bar{x} = 98.20^\circ$
 $s = 0.62^\circ$

$\alpha = 0.05$
 $\alpha/2 = 0.025$
 $z_{\alpha/2} = 1.96$

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\bar{x} - E < \mu < \bar{x} + E$$



★ if given a list of # and have to find everything to get your interval. No mean or standard deviation is given. When using a TI 84.

Jan 27-9:11 AM

Ex2b) A random sample of light bulbs had a mean life of $\bar{x} = 548$ hours with a standard deviation of $s = 2$ hours. Construct a 90% confidence interval for the mean life, μ , of all light bulbs of this type.

$n =$
 $\bar{x} =$
 $s =$

$\alpha =$
 $\alpha/2 =$
 $z_{\alpha/2} =$

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\bar{x} - E < \mu < \bar{x} + E$$

Jan 27-9:32 AM

★ If given the confidence interval and then asked to find \bar{x} and E, use these formulas.

Finding the Point Estimate and E from a Confidence Interval

sample mean \bar{x} is midway between the confidence intervals.

Point estimate of μ :

$$\bar{x} = \frac{(\text{upper confidence interval limit}) + (\text{lower confidence interval limit})}{2}$$

margin of error is $\frac{1}{2}$ the distance between the limits.

Margin of Error:

$$E = \frac{(\text{upper confidence interval limit}) - (\text{lower confidence interval limit})}{2}$$

Jan 27-9:16 AM

Ex3a

$$98.08^\circ < \mu < 98.32^\circ$$

$$\bar{x} = \frac{(\text{upper confidence interval limit}) + (\text{lower confidence interval limit})}{2}$$



$$\bar{x} = \frac{98.08^\circ + 98.32^\circ}{2} = 98.2^\circ$$

$$E = \frac{(\text{upper confidence interval limit}) - (\text{lower confidence interval limit})}{2}$$



$$E = \frac{98.32^\circ - 98.08^\circ}{2} = 0.12^\circ$$



Always check with the student administrator when you need help with a question. Please do not discuss it with anyone else.

Jan 27-9:18 AM

Ex3b)

Find the point estimate and E for the confidence interval. $543 < \mu < 553$

$$\bar{x} = \frac{(\text{upper confidence interval limit}) + (\text{lower confidence interval limit})}{2}$$

$$E = \frac{(\text{upper confidence interval limit}) - (\text{lower confidence interval limit})}{2}$$

Jan 27-9:41 AM

Try 1. If the confidence interval is $136 < \mu < 150$, what is the mean, \bar{x} and margin of error E?

Try 2: Find the margin of error in estimating the population mean and the confidence interval for the population mean μ for the following data:

Weights of eggs:

95% confidence interval
n = 49
x = 1.71 oz
s = 0.39 oz

Jan 27-10:06 AM

Ex4) Find the confidence interval when given a set of data:

p. 791 Data set 10: Find a confidence interval for the weight of Plain brown M&M using a graphing calculator with 95% confidence.

1. Find the mean and standard deviation.

- stats
- edit
- Enter the list in L1
- Stat
- Calc
- 1-vars stats
- enter
- enter

```
EDIT 1-VAR STATS
1-Var Stats
2-Var Stats
3-Freq/Inv
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7:QuarticReg
8:QuinticReg
9:Z-Test
10:T-Test
11:2Proportion
12:1Proportion
13:G-Test
14:ChiSqGTest
15:ChiSqTest
16:Y-Test
17:Y-Test
18:Y-Test
19:Y-Test
20:Y-Test
21:Y-Test
22:Y-Test
23:Y-Test
24:Y-Test
25:Y-Test
26:Y-Test
27:Y-Test
28:Y-Test
29:Y-Test
30:Y-Test
31:Y-Test
32:Y-Test
33:Y-Test
34:Y-Test
35:Y-Test
36:Y-Test
37:Y-Test
38:Y-Test
39:Y-Test
40:Y-Test
41:Y-Test
42:Y-Test
43:Y-Test
44:Y-Test
45:Y-Test
46:Y-Test
47:Y-Test
48:Y-Test
49:Y-Test
50:Y-Test
51:Y-Test
52:Y-Test
53:Y-Test
54:Y-Test
55:Y-Test
56:Y-Test
57:Y-Test
58:Y-Test
59:Y-Test
60:Y-Test
61:Y-Test
62:Y-Test
63:Y-Test
64:Y-Test
65:Y-Test
66:Y-Test
67:Y-Test
68:Y-Test
69:Y-Test
70:Y-Test
71:Y-Test
72:Y-Test
73:Y-Test
74:Y-Test
75:Y-Test
76:Y-Test
77:Y-Test
78:Y-Test
79:Y-Test
80:Y-Test
81:Y-Test
82:Y-Test
83:Y-Test
84:Y-Test
85:Y-Test
86:Y-Test
87:Y-Test
88:Y-Test
89:Y-Test
90:Y-Test
91:Y-Test
92:Y-Test
93:Y-Test
94:Y-Test
95:Y-Test
96:Y-Test
97:Y-Test
98:Y-Test
99:Y-Test
100:Y-Test
```

```
1-Var Stats
x̄=91.28181818
Σx=30.123
Σx²=27.546793
Sx=.0395169699
σx=.0389136219
n=33
```

Jan 27-11:01 AM

n =
x =
s =

α =
 $\alpha/2$ =
 $z_{\alpha/2}$ =

```
1-Var Stats
x̄=91.28181818
Σx=30.123
Σx²=27.546793
Sx=.0395169699
σx=.0389136219
n=33
```

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

*use S for σ

$$\bar{x} - E < \mu < \bar{x} + E$$

```
ZInterval
Inpt: Stats
σ: 0.03951
List: L1
Freq: 1
C-Level: .95
Calculate
```

```
ZInterval
(-.89934, .9263)
x̄=91.28181818
Sx=.0395169699
n=33
```

Jan 28-12:02 PM

You can also use TI 84 to check your intervals:

- stats
- test
- z-interval
- use stats when given n, s, \bar{x}
- enter mean
- use s instead of σ
- enter n
- confidence level
- calculate

Jan 28-2:05 PM