6-2

Estimating a Population Mean: Large Samples

- **n > 30**
  The sample must have more than 30 values.
- **Simple Random Sample**
  All samples of the same size have an equal chance of being selected.

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Definitions

- **Estimator**
  a formula or process for using sample data to estimate a population parameter
- **Estimate**
  a specific value or range of values used to approximate some population parameter
- **Point Estimate**
  a single value (or point) used to approximate a population parameter

The sample mean $\bar{x}$ is the best point estimate of the population mean μ.

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Definition

Confidence Interval

(or Interval Estimate)

a range (or an interval) of values **used to estimate** the true value of the population parameter

Lower # < population parameter < Upper #

As an example

Lower # < μ < Upper #

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Definition

Degree of Confidence

(level of confidence or confidence coefficient)

the probability 1 - α (often expressed as the equivalent percentage value) that is the relative frequency of times the confidence interval actually does contain the population parameter, assuming that the estimation process is repeated a large number of times

usually 90%, 95%, or 99%

(α = 10%), (α = 5%), (α = 1%)

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Interpreting a Confidence Interval

98.08° < μ < 98.32°

Correct: We are 95% confident that the interval from 98.08 to 98.32 actually does contain the true value of μ. This means that if we were to select many different samples of size 106 and construct the confidence intervals, 95% of them would actually contain the value of the population mean μ.

Wrong: There is a 95% chance that the true value of μ will fall between 98.08 and 98.32.

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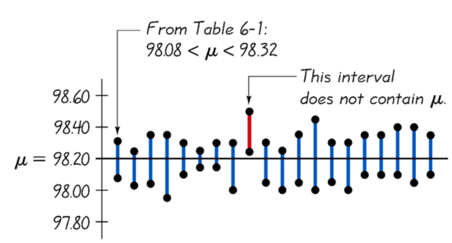
Confidence Intervals from 20 Different Samples



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Definition

Critical Value

the number on the borderline separating sample statistics that are likely to occur from those that are unlikely to occur. The number $z_{\alpha/2}$ is a critical value that is a z score with the property that it separates an area $\alpha/2$ in the right tail of the standard normal distribution.

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The Critical Value $z_{\alpha/2}$

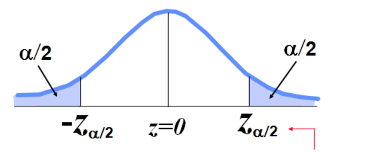

Figure 6-2

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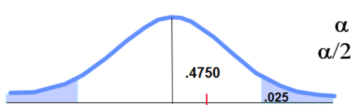
Ex1) Find the critical Value for $Z_{\alpha/2}$ that corresponds to the given degree of confidence.

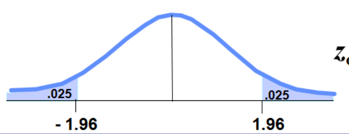
Finding $z_{\alpha/2}$ for 95% Degree of Confidence

$\alpha = 0.05$

$\alpha/2 = 0.025$



Use Table A-2 to find a z score of 1.96



$z_{\alpha/2} = \pm 1.96$

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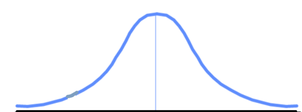
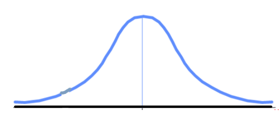
Ex1) Find the critical Value for $Z_{\alpha/2}$ that corresponds to the given degree of confidence.

b) 99%



c) 90%



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Degree of Confidence	α	Critical Value α/2
90 %	0.10	1.645
95 %	0.05	1.96
99 %	0.01	2.575

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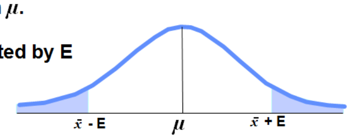
Definition

Margin of Error

is the maximum likely difference observed between sample mean $\bar{x}$ and true population mean μ.

denoted by E



$\bar{x} - E < \mu < \bar{x} + E$

lower limit — upper limit

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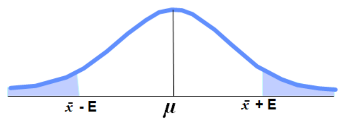
central limit theorem: $z = \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

* since n > 30, we will use *S* for our standard deviation not μ.

Margin of Error

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Formula 6-1



also called the maximum error of the estimate

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Confidence Interval (or Interval Estimate) for Population Mean μ

(Based on Large Samples: n >30)

$\bar{x} - E < \mu < \bar{x} + E$

$\mu = \bar{x} \pm E$

$(\bar{x} + E, \bar{x} - E)$

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Procedure for Constructing a Confidence Interval for μ

(Based on a Large Sample: $n > 30$)

1. Find the critical value $z_{\alpha/2}$ that corresponds to the desired degree of confidence.
2. Evaluate the margin of error $E = z_{\alpha/2} \cdot \sigma/\sqrt{n}$.
   If the population standard deviation σ is unknown, use the value of the sample standard deviation *s* provided that *n* > 30.
3. Find the values of $\bar{x}$ - E and $\bar{x}$ + E. Substitute those values in the general format of the confidence interval: $\bar{x} - E < \mu < \bar{x} + E$
4. Round using the confidence intervals roundoff rules.

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Round-Off Rule for Confidence Intervals Used to Estimate μ

1. When using the *original set of data*, round the confidence interval limits to one more decimal place than used in original set of data.
   ☆If given a list of # and have to find everything to get your interval. No mean or standard deviation is given. When using a TI 84.
2. When the original set of data is unknown and only the *summary statistics* *(n, x, s)* are used, round the confidence interval limits to the same number of decimal places used for the sample mean.

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Example: A study found the body temperatures of 106 healthy adults. The sample mean was 98.2 degrees and the sample standard deviation was 0.62 degrees. Find the margin of error E and the 95% confidence interval.

n = 106
$\bar{x}$ = 98.20°
s = 0.62°

α = 0.05
α /2 = 0.025
$z_{\alpha/2}$ = 1.96

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\bar{x} - E < \mu < \bar{x} + E$$

☆

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Ex2b) A random sample of light bulbs had a mean life of $\bar{x}$ = 548 hours with a standard deviation of s = 2 hours. Construct a 90% confidence interval for the mean life, μ, of all light bulbs of this type.

n =
$\bar{x}$ =
s =

α =
α /2 =
$z_{\alpha/2}$ =

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\bar{x} - E < \mu < \bar{x} + E$$

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☆ If given the confidence interval and then asked to find $\bar{x}$ and E, use these formulas.

Finding the Point Estimate and E from a Confidence Interval

sample mean $\bar{x}$ is midway between the confidence intervals.

Point estimate of μ:

$$\bar{x} = \frac{\text{(upper confidence interval limit)} + \text{(lower confidence interval limit)}}{2}$$

margin of error is ½ the distance between the limits.

Margin of Error:

$$E = \frac{\text{(upper confidence interval limit)} - \text{(lower confidence interval limit)}}{2}$$

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Ex3a

$98.08° < \mu < 98.32°$

$$\bar{x} = \frac{\text{(upper confidence interval limit)} + \text{(lower confidence interval limit)}}{2}$$

☆

$$E = \frac{\text{(upper confidence interval limit)} - \text{(lower confidence interval limit)}}{2}$$

☆

☆

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Ex3b)

Find the point estimate and E for the confidence interval. $543 < \mu < 553$

$$\bar{x} = \frac{\text{(upper confidence interval limit)} + \text{(lower confidence interval limit)}}{2}$$

$$E = \frac{\text{(upper confidence interval limit)} - \text{(lower confidence interval limit)}}{2}$$

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Try 1. If the confidence interval is $136 < \mu < 150$, what is the mean, $\bar{x}$ and margin of error E?

Try 2: Find the margin of error in estimating the population mean and the confidence interval for the population mean μ for the following data:

Weights of eggs:

95% confidence interval
n = 49
x = 1.71 oz
s = 0.39 oz

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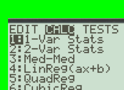
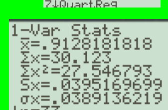
Ex4) Find the confidence interval when given a set of data:

p. 791 Data set 10: Find a confidence interval for the weight of Plain brown M&M using a graphing calculator with 95% confidence.

1. Find the mean and standard deviation.
- stats
- edit
- Enter the list in L_1
- Stat
- Calc
- 1-vars stats
- enter
- enter





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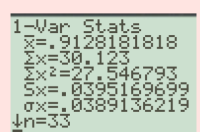
n =
$\bar{x}$ =
s =

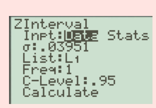
α =
α /2 =
$z_{\alpha/2}$ =

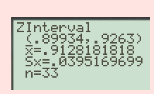


$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

*use S for σ

$$\bar{x} - E < \mu < \bar{x} + E$$





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You can also use TI 84 to check your intervals:

- stats
- test
- z-interval
- use stats when given n,s,x
- enter mean
- use s instead of σ
- enter n
- confidence level
- calculate

Jan 28-2:05 PM