Ch 6.5 Finding Population Proportions

1. The sample is a simple random sample.

2. The conditions for the binomial distribution are satisfied (See Section 4-3.)

3. The normal distribution can be used to approximate the distribution of sample proportions because $np \geq 5$ and $nq \geq 5$ are both satisfied.

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Notation for Proportions

$p =$ population proportion

$\hat{p} = \frac{x}{n}$ sample proportion

of x successes in a sample of size n

(pronounced 'p-hat')

The sample proportion $\hat{p}$ is the best point estimate of the population proportion p.

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Ex1a) A survey of 2450 golfers showed that 281 of them are left-handed. Find the point of estimate for p, the population proportion of golfers who are left-handed.

Ex1b) Find the point of estimate if 250 houses were surveyed and found that 62 people owned at least one gun.

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Margin of Error of the Estimate of p

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

Confidence Interval for Population Proportion

$$\hat{p} - E < p < \hat{p} + E$$

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Round-Off Rule for Confidence Interval Estimates of p

Round the confidence interval limits to

three significant digits.

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Ex2a) ***Misleading Responses:*** Do people lie about voting? In a survey 1002 people, 701 people said that they voted in the recent presidential election. The voting records show only 61% of eligible voters actually voted.

- Let's create a 95% confidence interval to determine the proportion of people who should have voted.

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} \qquad \hat{p} - E < p < \hat{p} + E$$

☆ $\hat{p} = \frac{x}{n} \quad \frac{701}{1002} = 0.6996$

☆ $\hat{q} = 1 - \hat{q} \quad = 0.3004$

☆ $E = 1.96\sqrt{\frac{(0.6996)(0.3004)}{1002}} = 0.0283855$

☆ $0.6996 - 0.0283855 < \mu < 0.6996 + 0.028355$

☆ $0.671 < \mu < 0.728$

Were they lying about who voted?

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Ex2b) A survey of 100 fatal accidents showed that 52 were alcohol related. Construct a 98% confidence interval for the proportion of fatal accidents that were alcohol related.

$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$ $\quad \hat{p} - E < p < \hat{p} + E$

☆ $\hat{p} = \frac{x}{n}$

☆ $\hat{q} = 1 - \hat{q}$

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Determining Sample Size

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

(solve for n by algebra)

$$n = \frac{(z_{\alpha/2})^2\,\hat{p}\hat{q}}{E^2}$$

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Sample Size for Estimating Proportion *p*

When an estimate of $\hat{p}$ is known:

$$n = \frac{(z_{\alpha/2})^2\,\hat{p}\hat{q}}{E^2}$$

When no estimate of p is known:

$$n = \frac{(z_{\alpha/2})^2\,0.25}{E^2}$$

$\hat{p}$	$\hat{q}$	$\hat{p}\hat{q}$
0.1	0.9	0.09
0.2	0.8	0.16
0.3	0.7	0.21
0.4	0.6	0.24
0.5	0.5	0.25
0.6	0.4	0.24
0.7	0.3	0.21
0.8	0.2	0.16
0.9	0.1	0.09

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Example: We want to determine, with a margin of error of four percentage points, the current percentage of U.S. households using e-mail. Assuming that we want 90% confidence in our results, how many households must we survey? A 1997 study indicates 16.9% of U.S. households used e-mail.

$$n = \frac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2}$$

[redacted]

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Example: We want to determine, with a margin of error of four percentage points, the current percentage of U.S. households using e-mail. Assuming that we want 90% confidence in our results, how many households must we survey? There is no prior information suggesting a possible value for the sample percentage.

$$n = \frac{[z_{\alpha/2}]^2 (0.25)}{E^2}$$

[redacted]

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Try: A pollster wishes to estimate the proportion of U.S. voters who favor capital punishment. How large a sample is needed in order to be 95% confident that the sample proportion will not differ from the true proportion by more than 2%?

☆ 2401

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Finding the Point Estimate and E from a Confidence Interval

Point estimate of $\hat{p}$:

$$\hat{p} = \frac{\text{(upper confidence interval limit)} + \text{(lower confidence interval limit)}}{2}$$

Margin of Error:

$$E = \frac{\text{(upper confidence interval limit)} - \text{(lower confidence interval limit)}}{2}$$

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Ex4) The following confidence interval represents the proportion of fatal accidents caused by the use cell phones while driving. Determine the margin of error and $\hat{p}$.

$$0.170 < p < 0.256$$

E

$\hat{p}$

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