Ch 9.3 Testing Population Means

State: 1. alpha
2. Ho and Ha
3. population mean

Plan: 1 sample mean t test
Check conditions: 1. Random sample
2. Less than 10% of population
3. n≤30 or normal (graph)

Do: find the test statistic: $t = \frac{\bar{x} - \mu_0}{\frac{s_x}{\sqrt{n}}}$

find the p-value: t- table using degree of freedom
or use the graphing calculator

Conclude: 1. reject or fail to reject Ho
2. p value < alpha
3. statement about the content

Jan 26-3:42 PM

Example 1: In an earlier example, a company claimed to have developed a new AAA battery that lasts longer than its regular AAA batteries. Based on years of experience, the company knows that its regular AAA batteries last for 30 hours of continuous use, on average. An SRS of 15 new batteries lasted an average of 33.9 hours with a standard deviation of 9.8 hours. Do these data give convincing evidence that the new batteries last longer on average?

STATE:

H_0: μ = 30 hours
H_a: μ > 30 hours

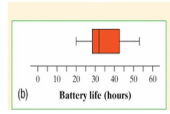
Test μ at 0.5 significance level.
where μ is the true mean lifetime of the new deluxe AAA batteries.

Plan: Perform a 1 sample mean t-test

Random: The company tested a simple random sample of 15 new AAA batteries.

10%: Because the batteries are being sampled without replacement, we need to check that there are at least 10(15) = 150 new AAA batteries. This seems reasonable to believe.

Normal/Large Sample: We don't know if the population distribution of battery lifetimes for the company's new AAA batteries is Normal. With such a small sample size (n = 15), we need to graph the data to look for any departures from Normality. **The boxplot shows slight right-skewness but no outliers. Because there isn't any strong skewness or outliers, we should be safe performing a test about the population mean lifetime μ.**



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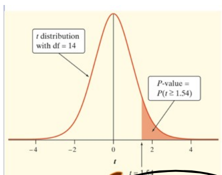
Do: $t = \frac{\bar{x} - \mu_0}{\frac{s_x}{\sqrt{n}}} = \frac{33.9 - 30}{\frac{9.8}{\sqrt{15}}} = 1.54$



Upper-tail probability p

df	.10	.05	.025
13	1.350	1.771	2.160
14	1.345	1.761	2.145
15	1.341	1.753	3.131
	80%	90%	95%

Confidence level C

The P-value is the probability of getting a result this large or larger in the direction indicated by H_a, that is, $P(t \geq 1.54)$.

✓ Go to the df = 14 row.

✓ Since the t statistic falls between the values 1.345 and 1.761, the "Upper-tail probability p" is between 0.10 and 0.05.

✓ The P-value for this test is between 0.05 and 0.10.

*calculator run a T-test
Stats, calc, t-test use, stats
p-value = .072

Conclude: Fail to Reject the Ho because 0.72 > α=0.05
We can't conclude the company's new AAA batteries last longer than 30 hrs on average.

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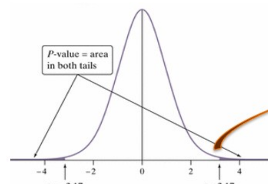
Example 2 What if you were performing a test of H_0: μ = 5 versus H_a: $\mu \neq 5$ based on a sample size of n = 37 and obtained t = −3.17? Since this is a two-sided test, you are interested in the probability of getting a value of t less than −3.17 or greater than 3.17. The figure below shows the desired P-value as an area under the t distribution curve with 36 degrees of freedom.



Upper-tail probability p

df	.005	.0025	.001
29	2.756	3.038	3.396
30	2.750	3.030	3.385
40	2.704	2.971	3.307
	99%	99.5%	99.8%

Confidence level C

Since df = 37 – 1 = 36 is not available on the table, move across the df = 30 row and notice that t = 3.17 falls between 3.030 and 3.385.
The corresponding "Upper-tail probability p" is between 0.0025 and 0.001. For this two-sided test, the corresponding P-value would be between 2(0.001) = 0.002 and 2(0.0025) = 0.005.

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Example 3: The level of dissolved oxygen (DO) in a stream or river is an important indicator of the water's ability to support aquatic life. A researcher measures the DO level at 15 randomly chosen locations along a stream. Here are the results in milligrams per liter:

State: We want to perform a test at the α = 0.05 significance level of

H_0: μ = 5
H_a: μ < 5

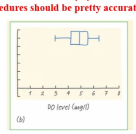
where μ is the actual mean dissolved oxygen level in this stream.

Plan: If conditions are met, we should do a one-sample t test for μ.

✓**Random:** The researcher measured the DO level at 15 randomly chosen locations.

✓**10%:** There is an infinite number of possible locations along the stream, so it isn't necessary to check the 10% condition.

Normal/Large Sample: We don't know whether the population distribution of DO levels at all points along the stream is Normal. With such a small sample size (n = 15), we need to graph the data to see if it's safe to use t procedures. **The boxplot, shows no outliers and is fairly symmetric. With no outliers or strong skewness, the t procedures should be pretty accurate.**



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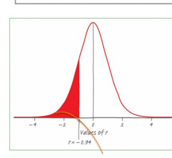
Do: We entered the data into our calculator and did 1-Var Stats (see screen shot).

```
NORMAL FLOAT AUTO REAL RADIAN CL
1-Var Stats
x̄=4.771333333
Σx=71.57
Σx²=353.8441
Sx=.9395961645
σx=.907736134
n=15
minX=2.87
↓Q1=4.13
```

Test Statistic $t = \frac{\bar{x} - \mu_0}{\frac{s_x}{\sqrt{n}}} = \frac{4.771 - 5}{\frac{0.9396}{\sqrt{15}}} = -0.94$

Using technology: We can find the exact P-value using a calculator: tcdf(lower: −100, upper: −0.94, df: 14) = 0.1816. or run t test using data



***P*-value** The P-value is the area to the left of t = –0.94 under the t distribution curve with df = 15 – 1 = 14. shows this probability. Using the table: Table B shows only areas in the upper tail of the distribution. Because the t distributions are symmetric, $P(t \leq -0.94) = P(t \geq 0.94)$. Search the df = 14 row of Table B for entries that bracket t = 0.94 (see the excerpt below). Because the observed t lies between 0.868 and 1.076, the P-value lies between 0.15 and 0.20.

Conclude: Fail to reject H_0. Since the P-value, is between 0.15 and 0.20 which is greater than our α = 0.05 significance level, we don't have enough evidence to conclude that the mean DO level in the stream is less than 5 mg/l.

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Example 6: Minitab output for a significance test and confidence interval based on the pineapple data is shown below. The test statistic and P-value match what we got earlier (up to rounding).

One-Sample T: Weight (oz)

```
Test of mu = 31 vs not = 31
Variable      N   Mean   StDev SE Mean        95% CI         T      P
Weight (oz)  50  31.935  2.394  0.339  (31.255, 32.616)  2.76  0.008
```

The 95% confidence interval for the mean weight of all the pineapples grown in the field this year is 31.255 to 32.616 ounces. We are 95% confident that this interval captures the true mean weight μ of this year's pineapple crop.

As with proportions, there is a link between a two-sided test at significance level α and a 100(1 – α)% confidence interval for a population mean μ.

For the pineapples, the two-sided test at α =0.05 rejects H_0: $\mu = 31$ in favor of H_a: $\mu \neq 31$. The corresponding 95% confidence interval does not include 31 as a plausible value of the parameter μ. In other words, the test and interval lead to the same conclusion about H_0. But the confidence interval provides much more information: *a set of plausible values for the population mean.*

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Confidence Intervals and Two-Sided Tests

The connection between two-sided tests and confidence intervals is even stronger for means than it was for proportions. That's because both inference methods for means use the standard error of the sample mean in the calculations.

Test statistic: $t = \dfrac{\bar{x} - \mu_0}{\frac{s_x}{\sqrt{n}}}$ Confidence interval: $\bar{x} \pm t^* \dfrac{s_x}{\sqrt{n}}$

✓ A two-sided test at significance level α (say, $\alpha = 0.05$) and a 100(1 – α)% confidence interval (a 95% confidence interval if $\alpha = 0.05$) give similar information about the population parameter.

✓ When the two-sided significance test at level α rejects H_0: $\mu = \mu_0$, the 100(1 – α)% confidence interval for μ will not contain the hypothesized value μ_0 .

✓ When the two-sided significance test at level α fails to reject the null hypothesis, the confidence interval for μ will contain μ_0 .

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