12-7 Counting Methods and Permutations

1 EXAMPLE Using a Tree Diagram

Suppose you have three shirts and two pair of pants that coordinate well. Make a tree diagram to find the number of possible outfits you have.

Shirts	Pants	Outfits
Shirt 1	Pants 1	Shirt 1, Pants 1
	Pants 2	Shirt 1, Pants 2
Shirt 2	Pants 1	Shirt 2, Pants 1
	Pants 2	Shirt 2, Pants 2
Shirt 3	Pants 1	Shirt 3, Pants 1
	Pants 2	Shirt 3, Pants 2

There are six possible outfits.

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1 a. Suppose you have two T-shirts and four pairs of shorts you could bring for gym class. Make a tree diagram to find the number of possible outfits for gym.

t-shirt 1 → shorts 1, shorts 2, shorts 3, shorts 4

t-shirt 2 → shorts 1, shorts 2, shorts 3, shorts 4

Total of 8 outcomes

b. Critical Thinking Would you want to use a tree diagram to find the number of outfits for five T-shirts and eight pairs of shorts? Explain.

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Recall that when one event does not affect the result of a second event, the events are *independent*. When events are independent, you can find the number of outcomes using the Multiplication Counting Principle.

Rule Multiplication Counting Principle

If there are m ways to make a first selection and n ways to make a second selection, there are $m \times n$ ways to make the two selections.

Example For five shirts and eight pairs of shorts, the number of possible outfits is $5 \cdot 8 = 40$.

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2 EXAMPLE Using the Multiplication Counting Principle

Travel On the map there are two tunnels for cars going from New Jersey to Manhattan, New York, and three bridges from Manhattan to Brooklyn, New York. How many routes using a tunnel and then a bridge are there from New Jersey to Brooklyn through Manhattan?

$2 \cdot 3 = 6$ ← routes from New Jersey to Brooklyn through Manhattan

routes by tunnel, from New Jersey to Manhattan; routes by bridge, from Manhattan to Brooklyn

There are six possible routes from New Jersey to Brooklyn.

2 Pizza At the neighborhood pizza shop, there are five vegetable toppings and three meat toppings for a pizza. How many different pizzas can you order with one meat and one vegetable topping?

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3 EXAMPLE Counting Permutations

Baseball How many different batting orders can you have with 9 baseball players?

There are 9 choices for the first batter, 8 for the second, 7 for the third, and so on.

$9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 362{,}880$ **Use a calculator.**

There are 362,880 possible batting orders.

*short cut- use factorials-!
on your calculator it is under probability button and scroll over.

3 Swimming A swimming pool has eight lanes. In how many ways can eight swimmers be assigned lanes for a race?

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In how many ways can you select a right, center, and left fielder from eight players on a baseball team? To answer this question, you need to find the number of permutations of 8 objects (players) arranged 3 at a time.

# of players you pick from for right field x # of players you can pick from for center field x # of players you can pick from for left field

Permutations - order matters.
You can't have someone play all 3 positions.

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Ex4a) 4 finalists and 2 are picked at a time.

ex) 1 queen and the other is the runner up

* Can think of the counting rule: 4 first choice and 3 to pick from the second choice.

Ex4b) You have 30 people in the class and 3 will be chosen for bell work. How many different ways can I select 3 people to go to the board.

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Definition **Permutation Notation**

The expression ${}_nP_r$ represents the number of permutations of n objects arranged r at a time.

Formula: ${}_nP_r = \dfrac{n!}{(n-r)!}$

Example ${}_8P_3$ represents 8 objects (players) chosen 3 at a time, or $8 \cdot 7 \cdot 6 = 336$.

$$\frac{8!}{(8-3)!} = \frac{8 \times 7 \times 6 \times \cancel{5 \times 4 \times 3 \times 2 \times 1}}{\cancel{5 \times 4 \times 3 \times 2 \times 1}} = 336$$

*calculator: 8 probability button nPr 3=

4 EXAMPLE **Using Permutation Notation**

Simplify ${}_7P_4$.

4 Simplify each expression.

a. ${}_9P_3$ b. ${}_7P_3$ c. ${}_5P_2$

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5 EXAMPLE **Real-World Problem Solving**

Computers Suppose you use six different letters to make a computer password. Find the number of possible six-letter passwords.

There are 26 letters in the alphabet. You are finding the number of permutations of 26 different letters arranged 6 at a time.

$${}_{26}P_6 = 26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \quad \textbf{Use a calculator.}$$
$$= 165{,}765{,}600$$

There are 165,765,600 six-letter passwords in which letters do not repeat.

5 a. Suppose your cousin needs to choose a four-digit number to use with a new debit card. Find the number of possible four-digit numbers without repeating a digit.

b. **Critical Thinking** Is a six-letter password or a six-digit number harder for someone to guess? Explain.

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