1.2 Inductive Reasoning

Goal: To use conductive reasoning to make conjectures.

WARM-UP EXERCISES

Evaluate the expression for the indicated value of *x*.

1. x^2, for $x = 1, -1, -2$

2. $(x + 2)(x - 2)$, for $x = 0, 1$

3. $\frac{2x}{3}$, for $x = 1, 3$.

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VOCABULARY

A **conjecture** is an unproven statement that is based on a pattern or observation.

Inductive reasoning is a process that involves looking for patterns and making conjectures.

A **counterexample** is an example that shows a conjecture is false.

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Make a Conjecture

Complete the conjecture.

Conjecture: The square of an even number is __?__.

To complete the conjecture:
Step 1: you must begin by writing several examples.
Step 2: Look at the solutions to draw a conclusion.
Step 3: Check for validity -- Can you find a counterexample?

$2^2 = 4$ $\quad 4^2 = 16$ $\quad 6^2 = 36$

$8^2 = 64$ $\quad 10^2 = 100$ $\quad 12^2 = 144$

Conjecture: The square of an even number is *even*.

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Complete the conjecture based on the pattern in the examples.

1. ***Conjecture:*** The sum of any two even numbers and any one odd number is __?__.

EXAMPLES

$2 + 4 + 5 = 11$ $\quad 4 + 18 + 1 = 13$ $\quad 6 + 8 + 7 = 21$

$10 + 12 + 11 = 33$ $\quad 2 + 2 + 27 = 31$ $\quad 10 + 20 + 21 = 51$

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Complete the conjecture.

Conjecture: The sum of the first n even positive integers is __?__.

Begin by writing several examples.

When $n = 1$
2 [redacted]

When $n = 2$
$2 + 4 = 6$ [redacted]

When $n = 3$
$2 + 4 + 6 = 12$ [redacted]

When $n = 4$
$2 + 4 + 6 + 8 = 20$ [redacted]

[redacted]

[redacted]

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Exercise for Example 2

Complete the conjecture based on the pattern in the examples.

2. ***Conjecture:*** The product of 5 and any even number is divisible by __?__.

EXAMPLES

$5 \times 2 = 10 =$ [redacted] $\quad 5 \times 4 = 20 =$ [redacted] $\quad 5 \times 6 = 30 =$ [redacted]

$5 \times 8 = 40 =$ [redacted] $\quad 5 \times 14 = 70 =$ [redacted] $\quad 5 \times 100 = 500 =$ [redacted]

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Find a Counterexample

Show the conjecture is false by finding a counterexample.

Conjecture: The difference of two even numbers is positive.

SOLUTION

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Show the conjecture is false by finding a counterexample.

3. The product of -1 and any number is negative.

4. Any number that is divisible by 5 is divisible by 10.

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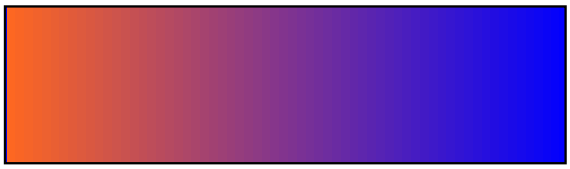
Find a Counterexample

Show the conjecture is false by finding a counterexample.

Conjecture: If a triangle is drawn inside of a circle with two corners of the triangle touching the circle, then the third corner of the triangle will also touch the circle.



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Show the conjecture is false by finding a counterexample.

5. If two circles touch each other, then one circle is inside the other.

Sample answer:

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Guided Practice

Vocabulary Check

1. Explain what a *conjecture* is.

2. How can you prove that a conjecture is false?

Skill Check

Complete the conjecture with *odd* or *even*.

3. ***Conjecture:*** The difference of any two odd numbers is _?_.

4. ***Conjecture:*** The sum of an odd number and an even number is _?_.

Show the conjecture is false by finding a counterexample.

5. Any number divisible by 2 is divisible by 4.

6. The difference of two numbers is less than the greater number.

7. A circle can always be drawn around a four-sided shape so that it touches all four corners of the shape.

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