MULTIPLE-CHOICE QUESTIONS

A calculator may not be used on the following questions.

All questions are for BC students only.

1. A particle moves in the xy-plane such that its position for time $t \geq 0$ is given by $x(t) = 3t^2 - 19t$ and $y(t) = e^{2t-7}$. What is the slope of the tangent to the path of the particle when $t = 4$?

(A) $-\frac{e}{28}$

(B) $-\frac{28}{e}$

(C) $\frac{e}{5}$

(D) $\frac{2e}{5}$

(E) $\frac{5}{2e}$

$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2e^{2t-7}}{6t-19}$ $\qquad \left.\frac{dy}{dx}\right|_{t=4} = \frac{2e}{5}$

2. The path of a particle in the xy-plane is given by the parametric equations $x(t) = \ln t$ and $y(t) = 5t^2 + 11$ for $t > 0$. An integral expression that represents the length of the path from $t = 2$ to $t = 6$ is

(A) $\int_2^6 \sqrt{\frac{1}{t^2} + 100t^2}\, dt$

(B) $\int_2^6 \sqrt{(\ln t)^2 + (5t^2 + 11)^2}\, dt$

(C) $\int_2^6 |5t^2 + 11 - \ln t|\, dt$

(D) $\int_2^6 \sqrt{1 + \frac{1}{t^2}}\, dt$

(E) $\int_2^6 \sqrt{1 + 100t^2}\, dt$

$\int_2^6 \sqrt{\left(\frac{1}{t}\right)^2 + (10t)^2}\, dt$

3. The position vector of a particle moving in the xy-plane is $\left(t - \cos t,\ t^3 - 12t\right)$ for $t \in [0, 2\pi]$. For what value of y does the path of the particle have a horizontal tangent?

(A) -16

(B) $\frac{\pi}{2}$

(C) $\frac{3\pi}{2}$

(D) 2

(E) 16

$3t^2 - 12 = 0$

$t = \pm 2$

$t = 2$

$[0, 2\pi]$

$y(2) = 8 - 24$

4. A plane curve has parametric equations $x(t) = t^2$ and $y(t) = t^4 + 3t^2$. An expression for the rate of change of the slope of the tangent to the path of the curve is
(A) $2t^2 + 3$
(B) $4t$
(C) $6t^2 + 3$
(D) $t^2 + 3$
(E) 2

$\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{4t^3+6t}{2t} = 2t^2+3$ $\quad \dfrac{d^2y}{dx^2} = \dfrac{4t}{2t} = 2$

5. A particle moves in the xy-plane for $t > 0$ so that $x(t) = t^2 - 4t$ and $y(t) = \ln t$. At time $t = 1$, the particle is moving
(A) to the right and up.
(B) to the left and up.
(C) to the left and down.
(D) to the right and down.
(E) in a direction that cannot be determined from the given information.

$y'(t) = \frac{1}{t}$ $\quad y'(1) = 1$ ↑
$x'(t) = 2t-4$ $\quad x'(1) = -2$ ←

6. The velocity vector of a particle moving in the xy-plane is $\left(\sqrt[3]{t},\ 6e^{2t-2}\right)$ for all real t. If the position of the particle at $t = 1$ is (0, 5), then the position vector of the particle is
(A) $\left(t^{\frac{4}{3}} - 1,\ 3e^{2t-1} + 2\right)$
(B) $\left(\frac{3}{4}t^{\frac{4}{3}},\ 3e^{2t-2}\right)$
(C) $\left(\frac{3}{4}t^{\frac{4}{3}},\ 6e^{2t-2} - 1\right)$
(D) $\left(\frac{1}{3t^{\frac{2}{3}}},\ 12e^{2t-2}\right)$
(E) $\left(\frac{3}{4}t^{\frac{4}{3}} - \frac{3}{4},\ 3e^{2t-2} + 2\right)$

$\int t^{1/3}dt = \frac{3}{4}t^{4/3} + C_1$
$\frac{3}{4}t^{4/3} + C_1 = 0$ at $t=1$
$C = -\frac{3}{4}$

$6\int e^{2t-2}dt = 3e^{2t-2} + C_2$
$3e^{2t-2} + C_2 = 5$ at $t=1$
$C = 2$

7. A particle moving in the xy-plane has position vector $\left(e^{2t}, \sqrt{t}\right)$ for $t \geq 0$. The acceleration vector of the particle is

(A) $\left(\frac{1}{2}e^{2t}, \frac{2}{3}t^{\frac{3}{2}}\right)$

(B) $\left(e^{2t}, -\frac{1}{4t^{\frac{3}{2}}}\right)$

(C) $\left(4e^{2t}, -\frac{1}{4t^{\frac{3}{2}}}\right)$

(D) $\left(2e^{2t}, \frac{1}{2\sqrt{t}}\right)$

(E) $\left(4e^{2t}, \frac{1}{4t^{\frac{3}{2}}}\right)$

$\frac{d}{dx}\left[e^{2t}\right] = 2e^{2t}$ $\quad \frac{d}{dx}\left[2e^{2t}\right] = 4e^{2t}$

$\frac{d}{dy}\left[t^{\frac{1}{2}}\right] = \frac{1}{2}t^{-\frac{1}{2}}$ $\quad \frac{d}{dy}\left[\frac{1}{2}t^{-\frac{1}{2}}\right] = -\frac{1}{4}t^{-\frac{3}{2}}$

A calculator may be used for the following questions.

8. A polar curve is given by $r = \frac{3}{2 - \cos\theta}$. The slope of the curve at $\theta = \frac{\pi}{2}$ is

(A) 0
(B) 0.5
(C) 0.75
(D) –0.75
(E) not defined.

$r = 3(2-\cos\theta)^{-1}$

$r' = \frac{-3\sin\theta}{(2-\cos\theta)^2}$

$y = r\sin\theta$ $\quad x = r\cos\theta$

$dy = r\cos\theta + \sin\theta r'$ $\quad dx = r(-\sin\theta) + \cos\theta r'$

$$\frac{dy}{dx} = \frac{\frac{3\cos\theta}{2-\cos\theta} + \frac{-3\sin^2\theta}{(2-\cos\theta)^2}}{\frac{-3\sin\theta}{(2-\cos\theta)} + \frac{-3\sin\theta\cos\theta}{(2-\cos\theta)^2}}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} \quad \frac{-\frac{3}{4}}{-\frac{3}{2}} = \frac{1}{2}$$

* can do $\frac{dy}{dx}$ on calcula by graphin

9. The position vector of a particle moving in the xy-plane is $\left(t^2, \sin t\right)$. What is the distance traveled by the particle from $t = 0$ to $t = \pi$?

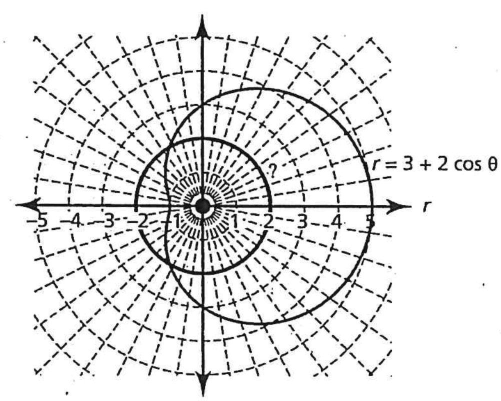
(A) 9.870
(B) 10.354
(C) 10.826
(D) 12.335
(E) 42.912

$$\int_0^{\pi} \sqrt{(2t)^2 + \cos^2 t}\; dt$$

10. The area inside the polar curve $r = 3 + 2\cos\theta$ is

(A) 9.425
(B) 18.850
(C) 28.274
(D) 34.558
(E) 69.115

$$2\left(\tfrac{1}{2}\right)\int_0^{\pi} (3+2\cos\theta)^2\, d\theta$$



11. A particle moves in the xy-plane so that its acceleration vector for time $t > 0$ is $\left(6t^2, \frac{20}{t}\right)$. If the velocity vector at $t = 1$ is $(5, 0)$, then how fast is the particle moving when $t = 3$?
(A) 36.069
(B) 54.410
(C) 58.299
(D) 61.088
(E) 75.972

$2t^3 + C = 5$ at $t=1$
$C = 3$

$20 \ln t + C = 0$
$C = 0$

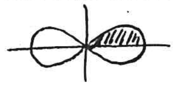
$\sqrt{(2(3)^3+3)^2 + (20\ln(3))^2}$

A calculator may not be used on the following questions.

12. A particle moves in the xy-plane so that its velocity for time $t \ge 0$ is given by the parametric equations $x'(t) = e^{2t}$ *and* $y'(t) = \sqrt{3t+1}$. An expression for the distance traveled by the particle on the interval $t \in [1, 5]$ is
(A) $\int_1^5 \left(e^{4t} + 3t + 1\right) dt$
(B) $\int_1^5 \sqrt{4e^{4t} + \frac{9}{4(3t+1)}}\, dt$
(C) $\int_1^5 \left(e^{2t} + \sqrt{3t+1}\right)^2 dt$
(D) $\int_1^5 \sqrt{e^{4t} + 3t + 1}\, dt$
(E) $\int_1^5 \left(\frac{1}{2}e^{2t} + \frac{2}{9}(3t+1)^{\frac{3}{2}}\right) dt$

$\int_1^5 \sqrt{(e^{2t})^2 + (\sqrt{3t+1})^2}\, dt$

13. The area enclosed inside the polar curve $r^2 = 10\cos(2\theta)$ is
(A) 10
(B) 5π
(C) 20
(D) 10π
(E) 25π



$4\left(\frac{1}{2}\right) \int_0^{\pi/4} 10 \cos(2\theta)\, d\theta$

$2(10)\left(\frac{1}{2}\right) \int_0^{\pi/2} \cos u \, du = 10\left[\sin u\right]_0^{\pi/2} = 1$

14. A particle moves in the xy-plane so that its velocity vector for time $0 \le t \le 10$ is $\left(\sqrt{100 - 10t}, 2t\right)$. Which one of the following statements is true about the particle when $t = 1$?
(A) The particle is slowing down.
(B) The particle is speeding up.
(C) The particle is at rest.
(D) The speed of the particle is $\sqrt{90} + 2$.
(E) The acceleration vector is $\left(\frac{1}{2\sqrt{90}}, 2\right)$.

$\|v(t)\| = \sqrt{(\sqrt{100 - 10t})^2 + (2t)^2}$
$= \sqrt{100 - 10t + 4t^2}$

$\frac{d}{dt}\|v(t)\| = \frac{1}{2}(100 - 10t + 4t^2)^{-1/2}(-10 + 8t)$
$= \frac{-5 + 4t}{(100 - 10t + 4t^2)^{1/2}}$

$\left.\frac{d}{dt}\right|_{t=1} = \frac{-1}{\sqrt{94}}$ IS NEGATIVE SO SLOWING DOWN