1. DIVERGES $\sum_{n=1}^{\infty} n^2$ — n^{th} TERM TEST $\lim_{n\to\infty} n^2 = \infty$

2. DIVERGES $\sum_{n=1}^{\infty} \frac{2n^2-1}{n^2+1}$ — n^{th} TERM TEST $\lim_{n\to\infty}(a_n) = 2$

3. CONVERGES $\sum_{n=0}^{\infty} \left(\frac{3^n}{4^n}\right)$ — GEOMETRIC $r = \frac{3}{4}$

4. CONVERGES $\sum_{n=1}^{\infty} \frac{1}{n^n}$ — $\left(\frac{1}{n}\right)^n \le \left(\frac{1}{2}\right)^n$ FOR $n \ge 2$
COMPARISON w/ GEOMETRIC

5. CONVERGES $\sum_{n=1}^{\infty} \frac{kn^2}{n!}$ — $\lim_{n\to\infty}\left|\frac{(n+1)^2}{(n+1)!}\cdot\frac{n!}{n^2}\right| = \lim_{n\to\infty}\left|\frac{n^2+2n+1}{n^3+n^2}\right| = 0$
RATIO TEST

6. CONVERGES $\sum_{n=2}^{\infty} \frac{1}{n^2-3}$ — $\lim_{n\to\infty} \frac{\frac{1}{n^2-3}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^2}{n^2-3} = 1$ limit is finite & positive and $\frac{1}{n^2}$ converges
LIMIT COMPARISON TEST

7. CONVERGES $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ — ALT. SERIES
① $\lim_{n\to\infty} \frac{1}{\ln n} = 0$
② $\frac{1}{\ln(n+1)} < \frac{1}{\ln n}$

8. CONVERGES $\sum_{n=0}^{\infty} \frac{3^{n+1}}{4^n} = 3\sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^n$ — GEOMETRIC $r = \frac{3}{4}$

9. CONVERGES $\sum_{n=0}^{\infty} \frac{1}{2^{n^2}}$ — $\left(\frac{1}{2}\right)^{n^2} \le \left(\frac{1}{2}\right)^n$ COMPARISON w/ GEOM.

10. CONVERGES $\sum_{n=2}^{\infty} (-1)^n \frac{1}{(\ln n)^3}$ — ALT. SERIES
① $\lim_{n\to\infty} \frac{1}{(\ln n)^3} = 0$
② $\frac{1}{(\ln(n+1))^3} \le \frac{1}{(\ln n)^3}$

11. DIVERGES $\sum_{n=3}^{\infty} \frac{1}{n-2}$ — $\lim_{b\to\infty}\int_3^b \frac{1}{x-2}\,dx = \lim_{b\to\infty} \ln|x-2|\Big]_3^b = \infty$
INTEGRAL TEST

12. CONVERGES $\sum_{n=0}^{\infty} 6^{-n} = \sum_{n=0}^{\infty}\left(\frac{1}{6}\right)^n$ — GEOMETRIC $r = \frac{1}{6}$

13. a) CONDITIONALLY CONVERGENT

$\sum_{n=2}^{\infty} \left|\frac{(-1)^n}{\ln n}\right| = \sum_{n=2}^{\infty} \frac{1}{\ln n}$ $\quad \frac{1}{\ln n} < \frac{1}{n}$ DIVERGE

$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ $\quad \lim_{n\to\infty} \frac{1}{\ln n} = 0$ $\quad \frac{1}{\ln(n+1)} < \frac{1}{\ln n}$ CONVERGE

b) $x=1$ ABSOLUTE CONVERGENCE

$\sum_{n=0}^{\infty} (-1)^n \left(\frac{x}{2}\right)^n$ $\quad x=1$ $\left|\frac{1}{2}\right|^n$ Geom $r=\frac{1}{2}$

$x=2$ Geom $r=1$

$r=\frac{3}{2}$

$x=2, x=3$ DIVERGES

c) CONVERGES ABSOLUTELY

$\sum_{n=2}^{\infty} \left|(-1)^n \left(\frac{n+1}{n^3}\right)\right|$ $\quad \sum_{n=2}^{\infty} \frac{n+1}{n^3}$ $\quad \lim_{n\to\infty} \frac{\frac{n+1}{n^3}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n+1}{n^3} \cdot \frac{n^3}{n} = 1$

LIMIT COMPARISON

M.C.

1. A $\quad -\frac{9}{2} \div 3 = -\frac{3}{2}$ $\quad \frac{27}{4} \cdot \left(-\frac{2}{9}\right) = -\frac{3}{2}$

2. C $\quad \frac{1}{n^4} < \frac{1}{1000}$ $\quad 1000 < n^4$ $\quad 6^4 = 1296$ so 5 terms

ALTERNATING so TERM BEFORE $< \frac{1}{1000}$

3. C

4. D $\quad \left(1 - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{7}\right) + \ldots = 1$

5. B $\quad$ A & B p-series $p<1$ DIVERGE B WORKS

D n^{th} TERM $\to \infty$ $\quad$ E compare to $\frac{1}{n}$ $\quad \frac{1}{\ln n} > \frac{1}{n}$ BOTH DIV.

6. B $\quad$ A, C, D ABS. CONV, E DIVERGE

7. D $\quad S = \frac{1}{1-\frac{2}{5}} = \frac{1}{3/5} = 5/3$

8. C $\quad \sum_{n=1}^{\infty} \frac{.5}{n} = .5 \sum_{n=1}^{\infty} \frac{1}{n}$

9. B $\quad |a_n|$ IS HARMONIC

10. E $\quad |r| < 1$ RATIO TEST CONVERGENCE