AP CALC BC CH. 9
PRACTICE PROBLEMS

1. E — FOR ABS. CONV. IGNORE ALT. $\therefore$ A) $\lim_{n\to\infty} = 1$ DIV B) $p = \frac{1}{2}$ DIV. C) $p = \frac{1}{2}$ DIV. D) $p = 1$ DIV E) $p = \frac{3}{2}$ CONV. ABS.

2. C — $\lim_{n\to\infty}\left|\frac{x^{n+1}}{n+1}\cdot\frac{n}{x^n}\right| = \lim_{n\to\infty}|x|\left|\frac{n}{n+1}\right| = |x|$ $|x|<1$
   if $x=-1$ $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ CONV if $x=1$ $\sum_{n=1}^{\infty}\frac{1}{n}$ DIV

3. D — $f(x) = x(\cos x - 1)$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots$
   $(\cos x - 1) = -\frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$
   $x(\cos x - 1) = -\frac{x^3}{2!} + \frac{x^5}{4!} - \frac{x^7}{6!}$

4. B — $g'(x) = \frac{1}{1+x^2}$ so anti-derivative $\sum_{k=0}^{\infty}(-1)^k\frac{x^{2k+1}}{2k+1}$

5. E — GEOMETRIC $|9-a|>5$ $9-a>5$ or $9-a<-5$; $4>a$ $14<a$

6. A — $\sum_{k=0}^{\infty}(-9)^k\frac{x^{4k}}{(2k)!} \to \sum_{k=0}^{\infty}(-1)^k(3^2)^k\frac{(x^2)^{2k}}{(2k)!} \to \sum_{k=0}^{\infty}(-1)^k\frac{(3x^2)^{2k}}{(2k)!}$

7. C — $f'(x) = -7 + 5x + \frac{9}{4}x^2 - 24x^3$ $f''(x) = 5 + \frac{9}{2}x - 72x^2$ $f'''(x) = \frac{9}{2} - 144x$ $f'''(0) = \frac{9}{2}$

8. D — I lim comparison w/ $p^{1/2}$ DIV II ALT. SERIES CONV III Geom $|r|>1$ DI

9. A — $\sum_{k=0}^{\infty}(-5)^k\frac{x^{3k+2}}{k!} \to (-1)^k\frac{5^k(x^3)^k\cdot x^2}{k!} \to (-1)^k\frac{x^2\cdot(5x^3)^k}{k!}$
   $x^2\sum_{k=0}^{\infty}\frac{(-5x^3)^k}{k!} = x^2e^{-5x^3}$

10. C — $\lim_{k\to\infty}\left|\frac{x^{2k+2}}{(k+1)4^{k+1}}\cdot\frac{k\cdot 4^k}{x^{2k}}\right| = \lim_{k\to\infty}\left|\frac{k}{k+1}\cdot\frac{x^2}{4}\right|$ $\left|\frac{x^2}{4}\right|<1$ $|x^2|<4$ $|x|<2$ RADIUS

11. D — $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$ $\sin(x^2) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!}$ $f'(x) = 2x - \frac{6}{3\cdot2\cdot1}x^5 + \frac{10}{5\cdot4\cdot3\cdot2}x^9$

12. C — A) DIV B) ABS. CONV (Geom) C) COND CONV D) DIV E) ABS CONV

13. A — $e^{-\frac{1}{2}x^2} = 1 + (-\tfrac{1}{2}x^2) + \frac{(-\frac{1}{2}x^2)^2}{2!} + \frac{(-\frac{1}{2}x^2)^3}{3!}$ $-\frac{\frac{1}{8}x^6}{6}$ $-\frac{1}{48}$

14. C — $\frac{2^7}{7!}$ is next term in ALT. SERIES so $|f(2) - T_6(2)| < \frac{2^7}{7!}$ A) & B) < C) GREATER D) IS NEG E) NO $< \frac{2^8}{8!}$

15. C — $T(x) = 3 + 2x + \frac{5}{2}x^2 + \frac{4}{6}x^3$ $T'(x) = 2 + 5x + 2x^2$ $T'(.1) = 2 + 5(.1) + 2(.1)^2$

CH. 9 PRACTICE PROBLEMS

FRQ

1. a)

$$\lim_{k\to\infty}\left|\frac{(k+2)x^{3(k+1)}}{(k+1)!}\cdot\frac{k!}{(k+1)x^{3k}}\right| = \lim_{k\to\infty}\left|\frac{k+2}{(k+1)(k+1)}\cdot\frac{x^3}{1}\right| = 0$$

$0<1$ $\therefore$ INT. CONV $(-\infty,\infty)$

b)

$$T_4(x) = 1 - 2x^3 + \frac{3x^6}{2!} - \frac{4x^9}{3!}$$

$$T_4'(x) = 0 - 3\cdot 2x^2 + \frac{6\cdot 3x^5}{2!} - \frac{9\cdot 4x^8}{3!}$$

$$w_3(x) = -6x^4 + 9x^7 - 6x^{10}$$

$$\sum_{k=0}^{\infty}(-1)^k\frac{(k+1)x^{3k}}{k!}$$

$$\sum_{k=1}^{\infty}(-1)^k\frac{3k(k+1)x^{3k-1}}{k!}$$

$w(x) = x^2 f'(x)$ $\quad \displaystyle\sum_{k=1}^{\infty}(-1)^k\frac{3k(k+1)x^{3k+1}}{k!}$

c)

$$T_4(.5) = 1 - 2\left(\tfrac{1}{2}\right)^3 + \frac{3\left(\frac{1}{2}\right)^6}{2!} - \frac{4\left(\frac{1}{2}\right)^9}{3!} = .772$$

$$T_4(1) = 1 - 2(1)^3 + \frac{3(1)^6}{2!} - \frac{4(1)^9}{3!} = -.167$$

d)

WHICH IS SMALLER, SINCE ALTERNATING SERIES

CHECK THE NEXT TERM

5^{th} TERM $\quad (-1)^4\dfrac{(5)x^{12}}{4!} \qquad \dfrac{5(.5)^{12}}{4!} \qquad \dfrac{5(1)^{12}}{4!}$

$.0000509 < .2083$