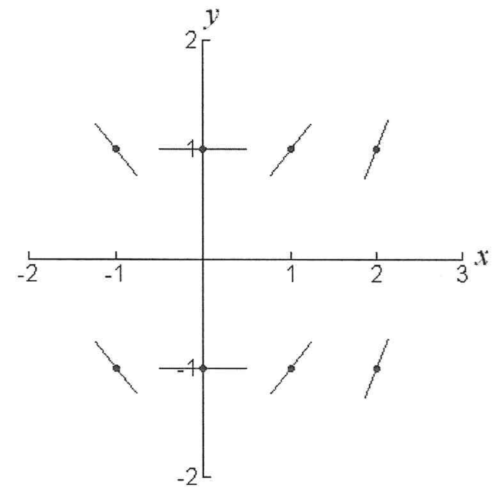
1. Given the differential equation: $\frac{dy}{dx} = \frac{x}{y^2}$

a. Sketch the slope field for the points: (1,±1), (2,±1),(−1, ±1), and (0,±1).  	+1 slopes of 0 along y-axis +1 relative steepness and correct slant of others
b. Find the general solution for the given differential equation. $\frac{dy}{dx} = \frac{x}{y^2}$ $y^2 dy = x\, dx$ $\int y^2 dy = \int x\, dx$ $\frac{1}{3}y^3 = \frac{1}{2}x^2 + C$ $y^3 = \frac{3}{2}x^2 + k$ $y = \sqrt[3]{\frac{3}{2}x^2 + k}$	+2 correct antiderivatives +1 correct use of constant of integration +1 correct solution for y
c. Find the solution of this differential equation that satisfies the initial condition $y(0) = 2$ $2 = \sqrt[3]{\frac{3(0)^2}{2} + k}$ $8 = k$ $y = \sqrt[3]{\frac{3x^2}{2} + 8}$	+1 correct substitution of point (0, 2) +1 correct solution

3. Given the differential equation $y' = \frac{2x}{y}$

a. Use Euler's Method to find the first three approximations of the particular solution passing through (1, 2). Use a step of $h = 0.2$ $y_1 = 2 + 0.2\left(\frac{2(1)}{2}\right) = 2.2$ $y_2 = 2.2 + 0.2\left(\frac{2(1.2)}{2.2}\right) = 2.418$ $y_3 = 2.418 + 0.2\left(\frac{2(1.4)}{2.418}\right) = 2.650$	+1 for 2.2 +1 for 2.418 +1 for 2.650
b. Find the general solution of the differential equation. $\frac{dy}{dx} = \frac{2x}{y}$ $y\,dy = 2x\,dx$ $\int y\,dy = \int 2x\,dx$ $\frac{1}{2}y^2 = x^2 + C$ $y^2 = 2x^2 + k$ $y = \pm\sqrt{2x^2 + k}$	+1: separates variables +2: correct antiderivatives +1: constant of integration
c. Find the particular solution of the differential equation that passes through (1, 2). $2 = \sqrt{2(1)^2 + k}$ $4 = 2 + k$ $k = 2$ $y = \sqrt{2x^2 + 2}$	+1: uses initial condition +1: solves for y

5. The number of bacteria in a culture is increasing according to the law of exponential growth. There are 150 bacteria in the culture after 3 hours and 400 bacteria after 5 hours.

a. Write an exponential growth model for the bacteria population. $150 = Ce^{3k}; \quad 400 = Ce^{5k}$ $Therefore,\ 400 = 150e^{-3k} \cdot e^{5k}$ $\frac{8}{3} = e^{2k}$ $\ln\frac{8}{3} = 2k$ $k = .4904$ $150 = Ce^{.4904(3)}$ $So,\ C = 34.447$ $y = 34.447e^{0.4904t}$	+1 correct value for k +1 correct value for C +1 exponential equation for y
b. Identify the initial population and the continuous growth rate. The initial population is approximately 34.447 and the continuous growth rate is 49.04%.	+1 initial population 34.447 +1 growth rate 49.04%
c. Use logarithms to determine after how many hours the bacteria count will be approximately 15,000? $15000 = 34.447e^{.4904t}$ $435.452 = e^{.4904t}$ $\ln 435.452 = .4904t$ $t \approx 12.391\ hours$	+ 1 set $y = 15,000$ +2 correct use of logs +1 correct answer