AP Calculus AB   3.1 – 3.6 Topics

1. How many critical points does the function have if $f'(x)=(x+2)^5(x-3)^4$?

   A. Two   B. Three   C. Four   D. Nine   E. None of these

   $x=-2$ $x=3$

2. What is the definition of extrema? MAX'S OR MIN'S

3. What is the definition of differentiation? BEING ABLE TO FIND OR SOLVE FOR A DERIVATIVE. IT IS NOT UNDEFINED

4. What is the definition of an open interval? AN INTERVAL THAT DOES NOT INCLUDE THE ENDPOINTS (a, b)

True or False

5. If the second derivative is equal to zero, there is a point of inflection.

   FALSE POSSIBLE P.O.I.

6. If there is a relative max or min on a closed interval, then the derivative is equal to zero.

   FALSE COULD BE UNDEFINED

7. If $f(a)=f(b)$ and the function is continuous on the closed interval, there must be a value of c such that $f'(c)=0$.

   FALSE, MUST BE DIFFERENTIABLE (a, b)

Critical Value Method

Where does the function have positive y – values, where does it have negative y – values?

8. $f(x)=-4(x-2)(x+1)$

-4	$-$	$-$	$-$
$x-2$	$-$	$-$	$+$
$x+1$	$-$	$+$	$+$

$-$ $\quad -1 \quad$ $+$ $\quad 2 \quad$ $-$

POS y-values $(-1, 2)$

NEG y-values $(-\infty, -1)$ $(2, \infty)$

9. $y=x^3-x$

   $x(x+1)(x-1)$

$x-1$	$-$	$-$	$-$	$+$
x	$-$	$-$	$+$	$+$
$x+1$	$-$	$+$	$+$	$+$

$-$ $\quad -1 \quad$ $+$ $\quad 0 \quad$ $-$ $\quad 1 \quad$ $+$

POS y-values $(-1, 0)$ $(1, \infty)$

NEG y-values $(-\infty, -1$ $(0, 1$

10. Sketch the function using the methods of curve sketching.

$f(x)=x^3-4x^2+4x$ $f(x) = x(x-2)^2$

$f'(x)= 3x^2-8x+4$

$f'(x) = (3x-2)(x-2)$

$3x-2$	$-$	$+$	$+$
$x-2$	$-$	$-$	$+$
$+$	$2/3$ $-$	2	$+$

$f''(x)= 6x-8$

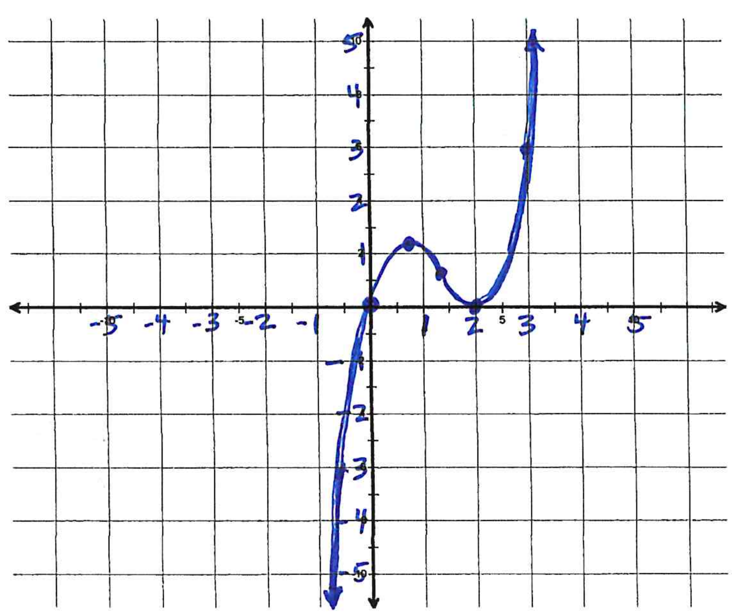
$f''(x)= 2(3x-4)$

$3x-4$	$-$	$+$
$-$	$4/3$	$+$

Intervals or x value	$f(x)$	$f'(x)$	$f''(x)$	Characteristics of Graph
$(-\infty, 2/3)$		$+$	$-$	inc CD
$x = 2/3$	$32/27$			
$(2/3, 4/3)$		$-$	$-$	dec CD
$x = 4/3$	$16/27$			
$(4/3, 2)$		$-$	$+$	dec CU
$x = 2$	0			
$(2, \infty)$		$+$	$+$	inc CU

y - Intercept $(0,0)$

Zero(s) $0, 2$ (d.z.)



$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = -\infty$

* CHANGE SCALE

11. Sketch the function using the methods of curve sketching.

$y = 3\sin(2t)$ On the Interval $[-\pi, \pi]$

$y'(t) = 6\cos(2t)$

$\cos u = 0$
$\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$

$\cos(2t)$: + | − | + | − | +

$-\pi$, $-\frac{3\pi}{4}$, $-\frac{\pi}{4}$, $\frac{\pi}{4}$, $\frac{3\pi}{4}$, π

$y''(t) = -12\sin(2t)$

$\sin(2t) = 0$: $0, \pm\pi, \pm 2\pi$

-12: − | − | − | −

$\sin(2t)$: + | − | + | −

$-\pi$ − $-\frac{\pi}{2}$ + 0 − $\frac{\pi}{2}$ + π

y - Intercept $(0,0)$

Intervals or x value	$f(x)$	$f'(x)$	$f''(x)$	Characteristics of Graph
$x=-\pi$	0			
$(-\pi, -\frac{3\pi}{4})$		+	−	inc CD
$x=-\frac{3\pi}{4}$	3			local max
$(-\frac{3\pi}{4}, -\frac{\pi}{2})$		−	−	dec CD
$x=-\frac{\pi}{2}$	0			POT
$(-\frac{\pi}{2}, -\frac{\pi}{4})$		−	+	dec CU
$x=-\frac{\pi}{4}$	−3			local min
$(-\frac{\pi}{4}, 0)$		+	+	inc CU
$x=0$	0			POI
$(0, \frac{\pi}{4})$		+	−	inc CD
$x=\frac{\pi}{4}$	3			local max
$(\frac{\pi}{4}, \frac{\pi}{2})$		−	−	dec CD
$x=\frac{\pi}{2}$	0			POI
$(\frac{\pi}{2}, \frac{3\pi}{4})$		−	+	dec CU
$x=\frac{3\pi}{4}$	−3			local min
$(\frac{3\pi}{4}, \pi)$		+	+	inc CU
$x=\pi$	0			

Zero(s)

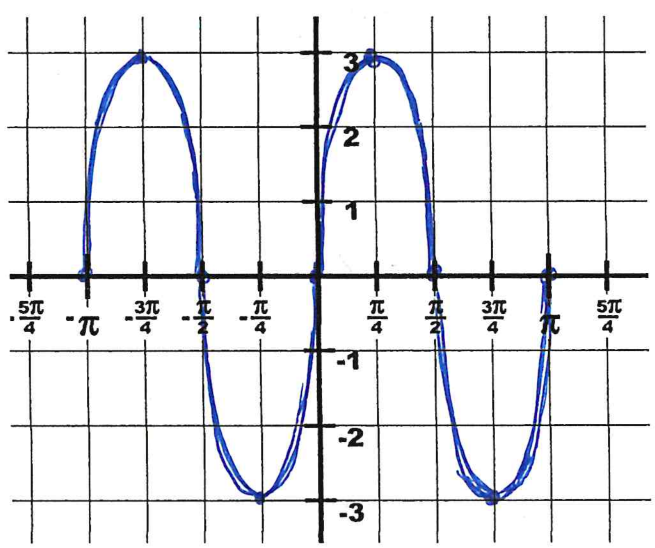
$\sin u = 0$
$0, \pm\pi, \pm 2\pi$

$(0,0)$
$(-\pi,0)$ $(-\frac{\pi}{2},0)$
$(\frac{\pi}{2},0)$ $(\pi,0)$

$\lim_{x\to\infty} y$ DNE

OSCILLATES BETWEEN
−3 AND 3

$\lim_{x\to-\infty} y$ DNE



12. Sketch the function using the methods of curve sketching.

$$f(x)=\frac{4x^2-8x}{2x^2-8}$$ *(hint: Simplify the function before finding the derivative)*

$f(x)=\frac{2x(x-2)}{(x-2)(x+2)}$

$\frac{dy}{dx}=\frac{4}{(x+2)^2}$

C.V. $x=-2$

ALWAYS POSITIVE

Intervals or x value	$f(x)$	$f'(x)$	$f''(x)$	Characteristics of Graph
$(-\infty,-2)$		+	+	inc C.U.
$x=-2$	undef			V.A.
$(-2,\infty$		+	−	inc C.D.

* hole in the graph @ $x=2$

$\frac{d^2y}{dx^2}=\frac{-8}{(x+2)^3}$

-8 − ; −

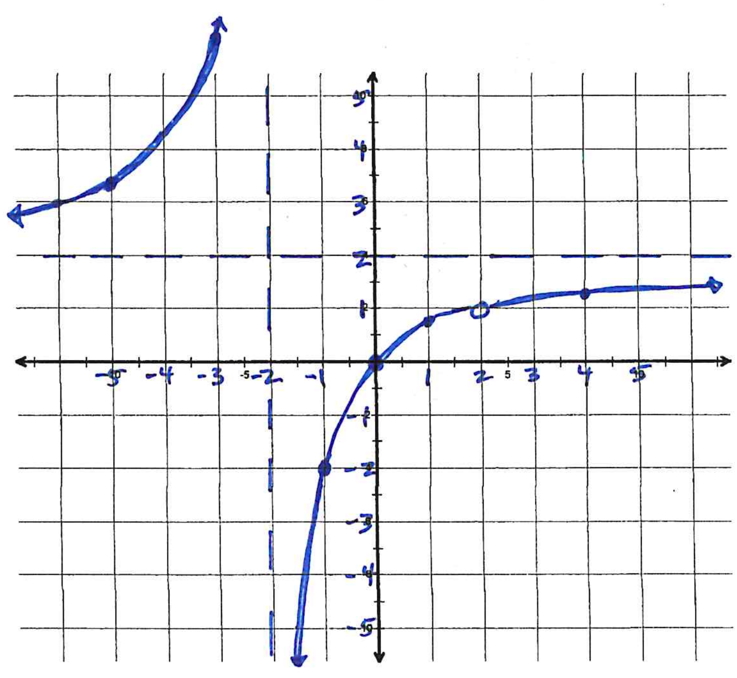
$(x+2)^3$ − + +

\+ −2 −

y - Intercept $(0,0)$

Zero(s) $(0,0)$

$\lim_{x\to\infty} f(x) = 2$



$\lim_{x\to-\infty} f(x) = 2$